

Aristotle on continuity of time in *Physics* VI 2

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Abstract

In *Physics*, 233 Aristotle proves that “all time is continuous” and defines a sequence of points that divide “the time ZH ” into “infinitely divisible parts”. In his proof, Aristotle applies basically the same mathematical method as that used in The Achilles Paradox. We present a mathematical formula that captures the essence of both the Achilles Paradox and Aristotle’s proof.

The Achilles Paradox

“The second is the so-called ‘Achilles’, and it amounts to this,
(1) that in a race the quickest runner can never overtake the
slowest,

(2) since the pursuer must first reach the point whence the pursued
started, so the slower must always hold a lead.

(3) This argument is the same in principle as that which depends
on bisection,

(4) though it differs from it in that the added magnitudes
[μεγεθος] are not divided into halves [διχα]”.

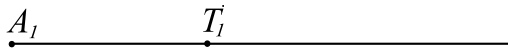
Aristotle, *Physics* 239b, after [Kirk et al.], p. 272.

The Achilles Paradox

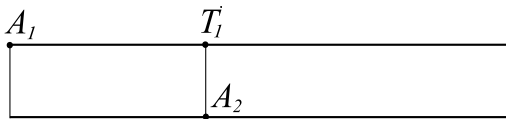
"The [second] argument was called 'Achilles', accordingly, from the fact that Achilles was taken [as a character] in it, and the argument says that it is impossible for him to overtake the tortoise when pursuing it. For in fact it is necessary that what is to overtake [something], before overtaking [it], first reach the limit from which what is fleeing set forth. In [the time in] which what is pursuing arrives at this, what is fleeing will advance a certain interval, even if it is less than that which what is pursuing advanced And in the time again in which what is pursuing will traverse this [interval] which what is fleeing advanced, in this time again what is fleeing will traverse some amount And thus in every time in which what is pursuing will traverse the [interval] which what is fleeing, being slower, has already advanced, what is fleeing will also advance some amount".

Simplicius, *On Aristotle's Physics* 6, 1014.10, translated by D. Konstan.

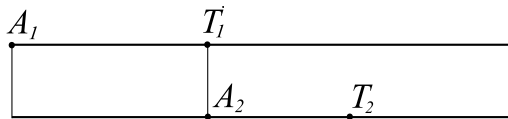
Aristotle, *Physics*, 239b



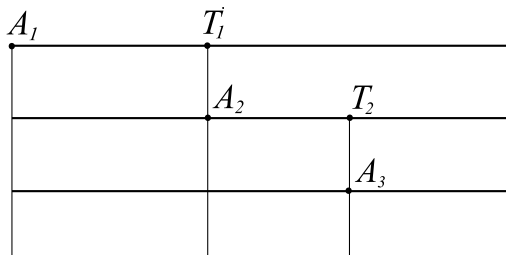
Aristotle, *Physics*, 239b



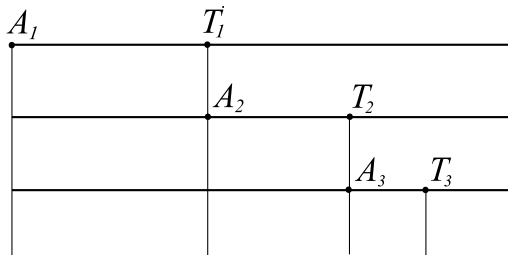
Aristotle, *Physics*, 239b



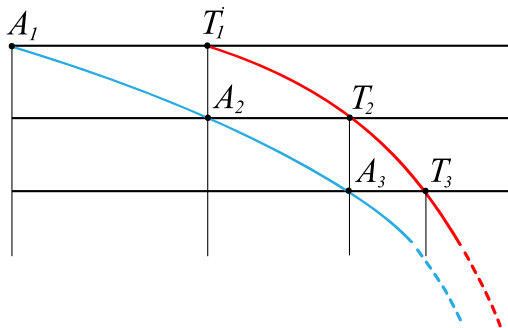
Aristotle, *Physics*, 239b



Aristotle, *Physics*, 239b



Aristotle, *Physics*, 239b



Continuum by Aristotle

Aristotle offers a triple characteristics of a continuum:

- (1) it is 'composed of parts', it is 'divisible' into parts,
- (2) 'the touching limits' of these parts are 'one and the same and are contained in each other',
- (3) these parts are 'infinitely divisible'.

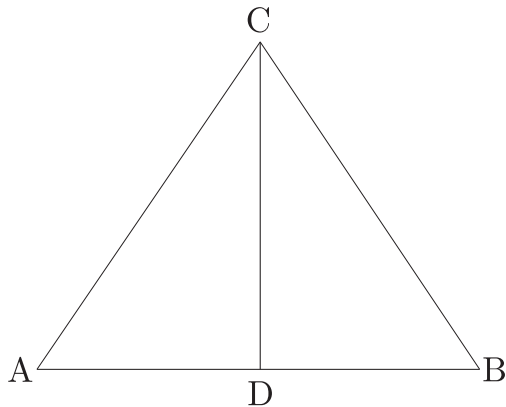
Aristotle, *Physics*, 231b, 227a, 231b.

Dichotomy by Euclid

"To cut a given straight line in half [$\delta\iota\chi\alpha$]".
Elements, I.10, translated by R. Fitzpatrick.

"To bisect a finite straight line". (T. Heath)

Dichotomy by Euclid



Dichotomy by Euclid

$$\begin{cases} D_0 = D, \\ D_{n+1} = \delta(D_n B), \text{ where } \delta \text{ stands for dichotomy.} \end{cases}$$

“[...] suppose that A is quicker and B slower, and that $[\alpha]$ the slower has traversed the magnitude GD in the time ZH. Now it is clear that $[\beta]$ the quicker will traverse the same magnitude in less time than this: let us say in the time ZO. Again, since the quicker has passed over the whole [GD] D in the time ZO, $[\gamma]$ the slower will in the same time pass over GK, say, which is less than GD. And since $[\alpha_1]$ B, the slower, has passed over GK in the time ZO, $[\beta_1]$ the quicker will pass over it in less time: so that the time ZO will again be divided. And if this is divided $[\gamma_1]$ the magnitude GK will also be divided just as GD was: and again, if the magnitude is divided, the time will also be divided. And we can carry on this process for ever, taking the slower after the quicker and the quicker after the slower alternately, and using what has been demonstrated at each stage as a new point of departure: for the quicker will divide the time and the slower will divide the length. If, then, this alternation always holds good, and at every turn involves a division, it is evident that **all time must be continuous**".

$$\alpha \quad GD = ZH\omega_B,$$

$$\beta \quad GD = ZO\omega_A,$$

$$\gamma \quad GK = ZO\omega_B.$$

$$\alpha \quad GD = ZH\omega_B,$$

$$\beta \quad GD = ZO\omega_A,$$

$$\gamma \quad GK = ZO\omega_B.$$

$$\alpha_1 \quad GK = ZO\omega_B,$$

$$\beta_1 \quad GK = ZO_1\omega_A,$$

$$\gamma_1 \quad GK_1 = ZO_1\omega_B.$$

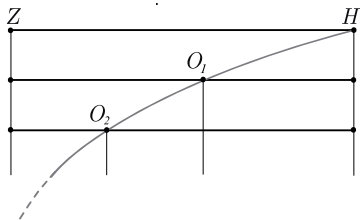
"the quicker $[\omega_A]$ will divide the time and the slower $[\omega_B]$ will divide the length"

$$\alpha \quad GD = ZH\omega_B,$$

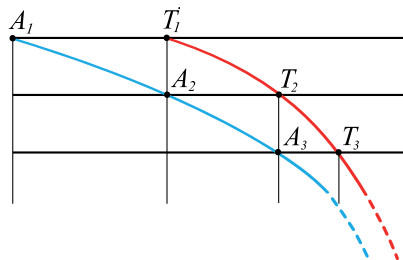
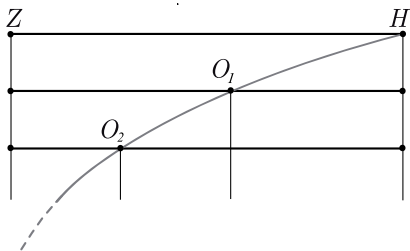
$$\beta \quad GD = ZO\omega_A, \quad O = \beta(GD)$$

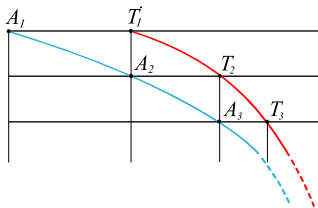
$$\gamma \quad GK = ZO\omega_B, \quad K = \gamma(ZO).$$

Let β stand for an operation that determines point O, that is to say $O = \beta(GD)$. Let γ stand for yet another operation that determines point K, $K = \gamma(ZO)$.



$$\begin{cases} O_0 = H, & K_0 = D, \\ O_{n+1} = \beta(GK_n), & K_{n+1} = \gamma(ZO_{n+1}). \end{cases} \quad (*)$$



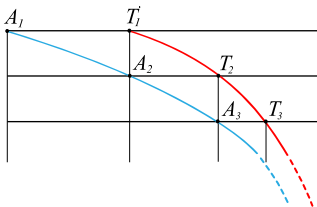


$$\beta \quad A_1 T_1 = t_1 \omega_A,$$

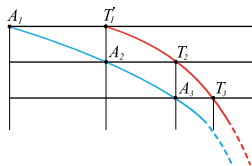
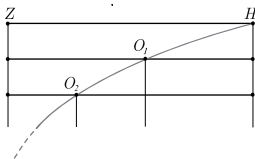
$$\gamma \quad T_1 T_2 = t_1 \omega_B.$$

Since we treat β and γ as operations, we can rewrite the above formule as follows:

$$\begin{cases} t_1 = \beta(A_1 T_1), \\ T_2 = \gamma(t_1). \end{cases}$$



$$\begin{cases} d_1 = A_1 T_1, \\ t_n = \beta(d_n), \quad d_{n+1} = \gamma(t_n). \end{cases} \quad (**)$$



$$\begin{cases} d_1 = A_1 T_1, \\ t_n = \beta(d_n), \end{cases} \quad d_{n+1} = \gamma(t_n). \quad (**)$$

$$\begin{cases} O_0 = H, \\ O_{n+1} = \beta(GK_n), \end{cases} \quad \begin{cases} K_0 = D, \\ K_{n+1} = \gamma(ZO_{n+1}). \end{cases} \quad (*)$$