Strukturalna poprawność argumentu.

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1. Definition of argument and further notions.

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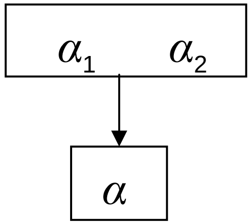
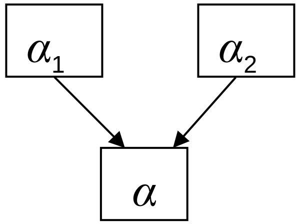
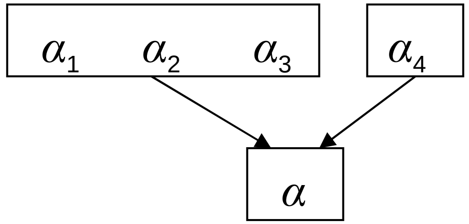
- Argument = konkluzja + przesłanki.
- Przesłanki mogą wspierać konkluzję:



- **łącznie (zespołowo, szeregowo)**

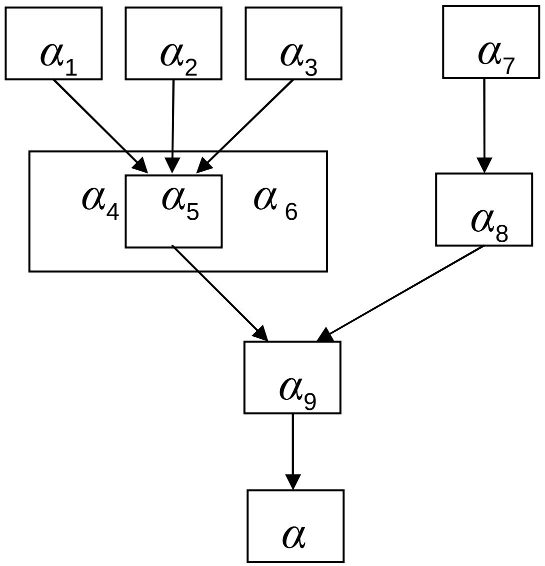


- **rozłącznie (rozdzielnie, równolegle)**



- **w sposób mieszany**

- Czasami przesłanki wspierają inne przesłanki:



- Let **S** be a set of sentences of a given language.
- Let $\mathbf{A} = < A_1, A_2, \ldots, A_{n_{\mathrm{A}}} >$ be a finite sequence of non-empty, finite relations defined on the set $P_{fin}(\mathbf{S}) \times \mathbf{S}$.

Thus $A_m = \{< P_m^1, \alpha_m^1 >, < P_m^2, \alpha_m^2 >, \ldots, < P_m^{i_m}, \alpha_m^{i_m} >\}$ for $m \leq n_{\mathbf{A}}$.

Def. 1.

$\mathbf{A}$ is ***an argument*** iff the following conditions hold:

(*i*) $\alpha_1^1 = \alpha_1^2 = \ldots = \alpha_1^{i_1}$ (*i.e.* for $m = 1$);

(*ii*) $\forall j \leq i_m \exists k \; \alpha_m^j \in P_{m-1}^k$ for $2 \leq m \leq n_{\mathrm{A}}$.

Further definitions.

- Assume that $\mathbf{A}$ = $<A_1, A_2, \ldots, A_{n_\mathrm{A}}>$ is an argument.

Def. 2.

The final conclusion of $\mathbf{A}$ is the sentence:

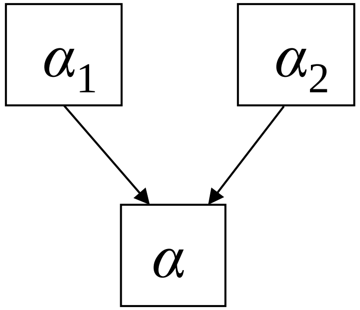
$$\alpha_1^1 = \alpha_1^2 = \ldots = \alpha_1^{i_1}$$

Def. 3.

A sentence is ***a premise*** of $\mathbf{A}$ iff it is an element of a set belonging to the domain of some of relations:

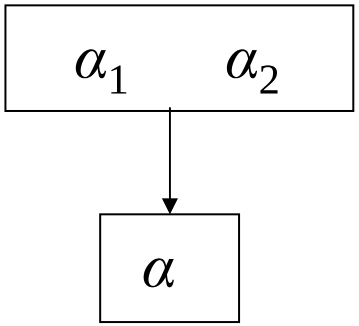
$$A_1, A_2, \ldots, A_{n_\mathrm{A}}.$$

Examples



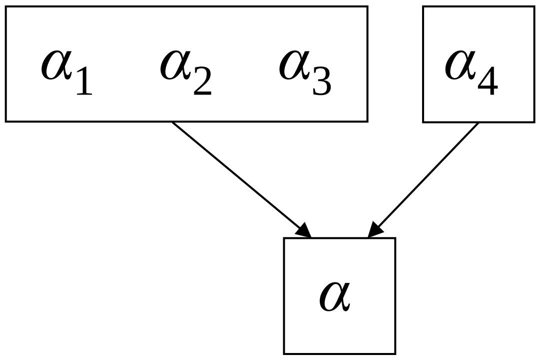
$$<\{<\{\alpha_1\}, \alpha >, <\{\alpha_2\}, \alpha >\}>$$

convergent argument

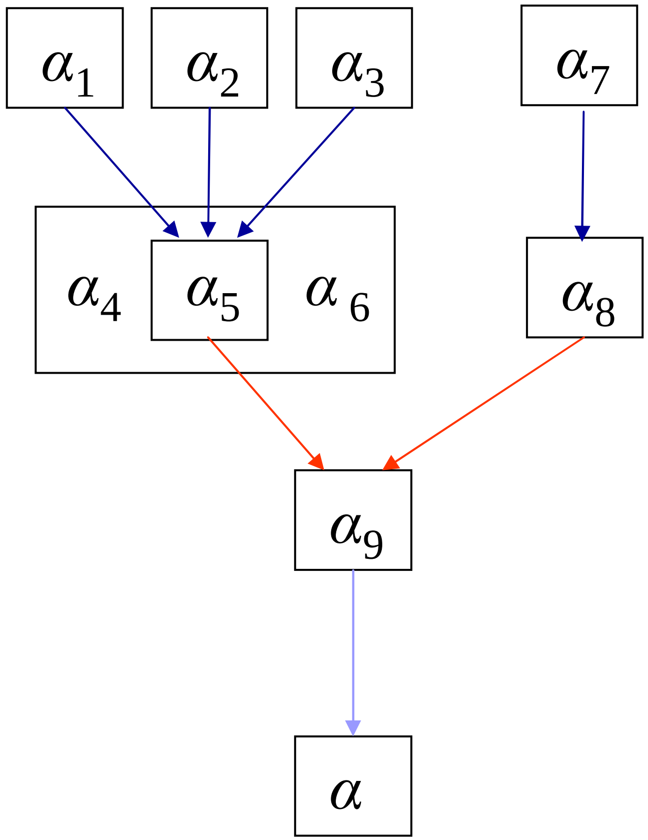


$$<\{<\{\alpha_1, \alpha_2\}, \alpha >\}>$$

linked argument



$$<\{<\{\alpha_1, \alpha_2, \alpha_3\}, \alpha >, <\{\alpha_4\}, \alpha >\}>$$



$$<\{<\{\alpha_9\},\alpha>\},$$
$$\{<\{\alpha_4,\alpha_5,\alpha_6\},\alpha_9>,<\{\alpha_8\},\alpha_9>\},$$
$$\{<\{\alpha_1\},\alpha_5>,<\{\alpha_2\},\alpha_5>,<\{\alpha_3\},\alpha_5>\},<\{\alpha_7\},\alpha_8>\}>.$$

$$\mathbf{A} = < A_1, A_2, \ldots, A_{n_\mathrm{A}} >$$

Def. 4.

The final argument of $\mathbf{A}$ is the one-element sequence $<A_1>$.

Def. 5.

The $\boldsymbol{m}$***-th level*** of $\mathbf{A}$ is the relation A_m (for $m \leq n_\mathrm{A}$).

Def. 6.

An argument $<\{<P, \beta>\}>$ is ***an atomic argument*** of $\mathbf{A}$ iff there exists $m \leq n_\mathrm{A}$ such that $<P, \beta> \in A_m$.

Def. 7.

An argument is ***direct*** iff
it consists of one level only.

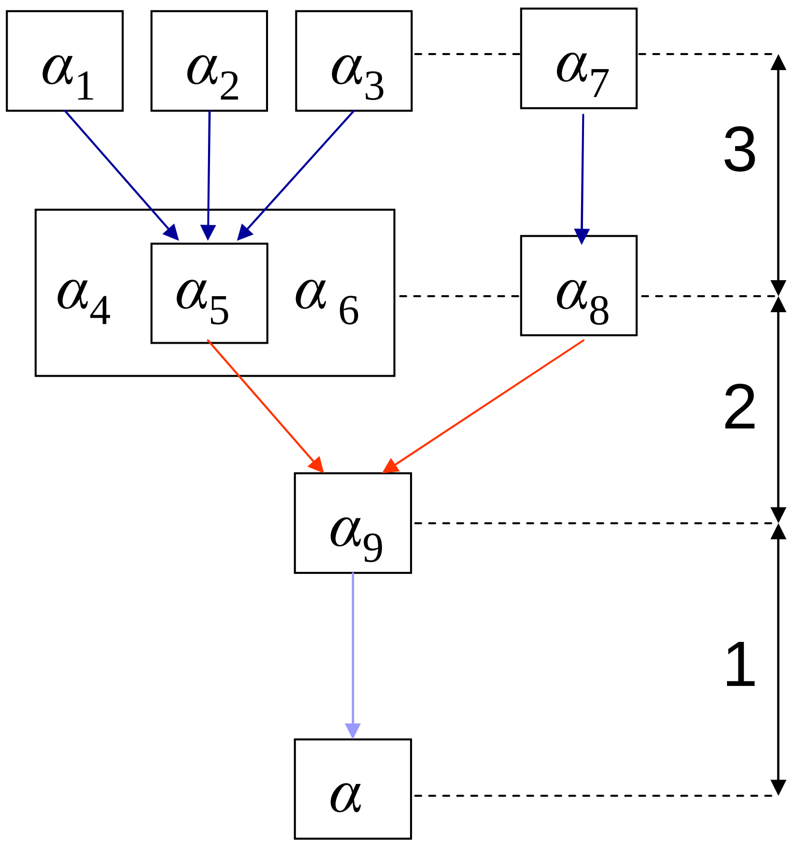
Def. 8.

A sentence is ***an intermediate conclusion*** of $\mathbf{A}$ iff
it belongs to the counterdomain of some of its levels,
which are higher then 1.

Def. 9.

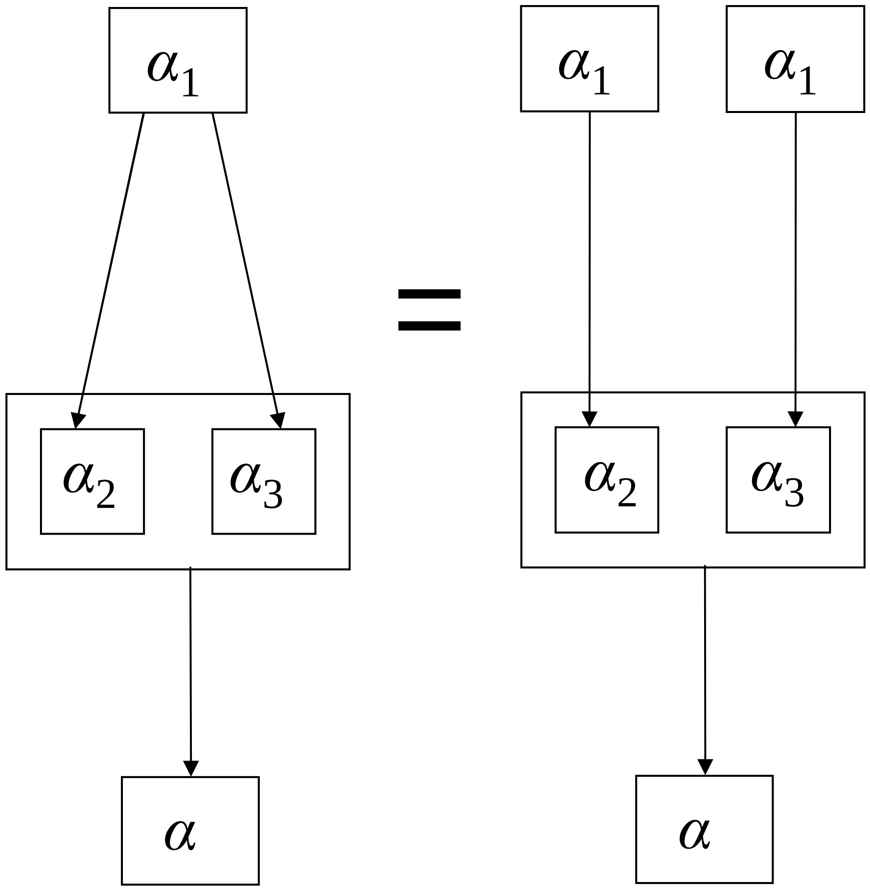
A sentence is ***a first premise*** of $\mathbf{A}$ iff
— it belongs to an element of the domain of $A_{n_{\mathrm{A}}}$.
or
— it belongs to an element of the domain of A_m (for $m < n_{\mathrm{A}}$),
but it does not belong to the counterdomain of A_{m+1}.

Examples

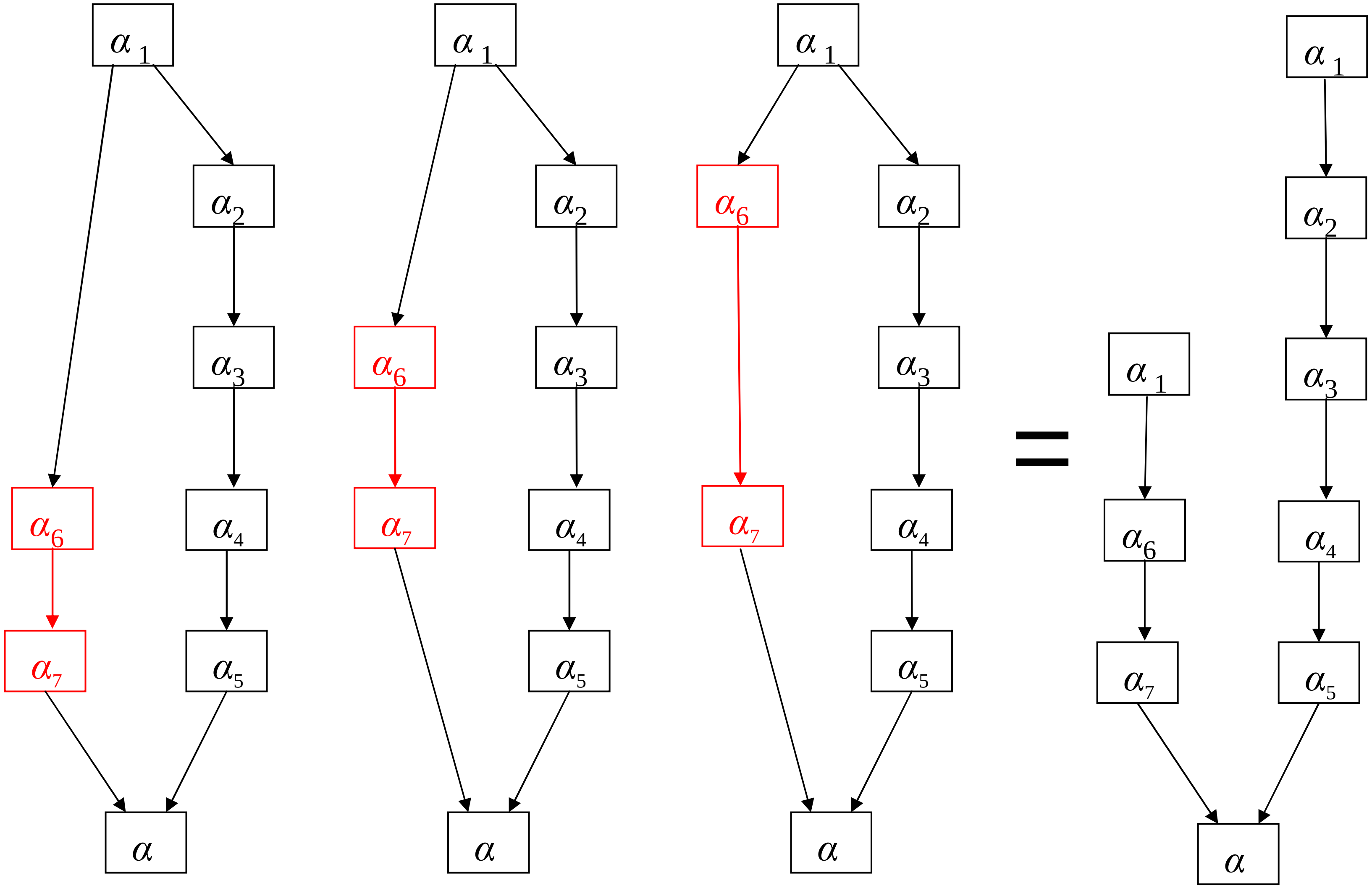


- final argument
- level of argument
- atomic argument
- direct argument
- intermediate conclusion
- first premise

$$< \{<\{\alpha_9\}, \alpha>\},$$
$$\{<\{\alpha_4, \alpha_5, \alpha_6\}, \alpha_9>, <\{\alpha_8\}, \alpha_9>\},$$
$$\{<\{\alpha_1\}, \alpha_5>, <\{\alpha_2\}, \alpha_5>, <\{\alpha_3\}, \alpha_5>\}, <\{\alpha_7\}, \alpha_8>\}>.$$



$$<\{<\{\alpha_2, \alpha_3\}, \alpha>\}, \{<\{\alpha_1\}, \alpha_2>, <\{\alpha_1\}, \alpha_3>\}>$$



α 1
α2
α3
α4
α5
α6
α7
α
=

Def. 10

The domain of **A** is the set of all the premises of **A**.

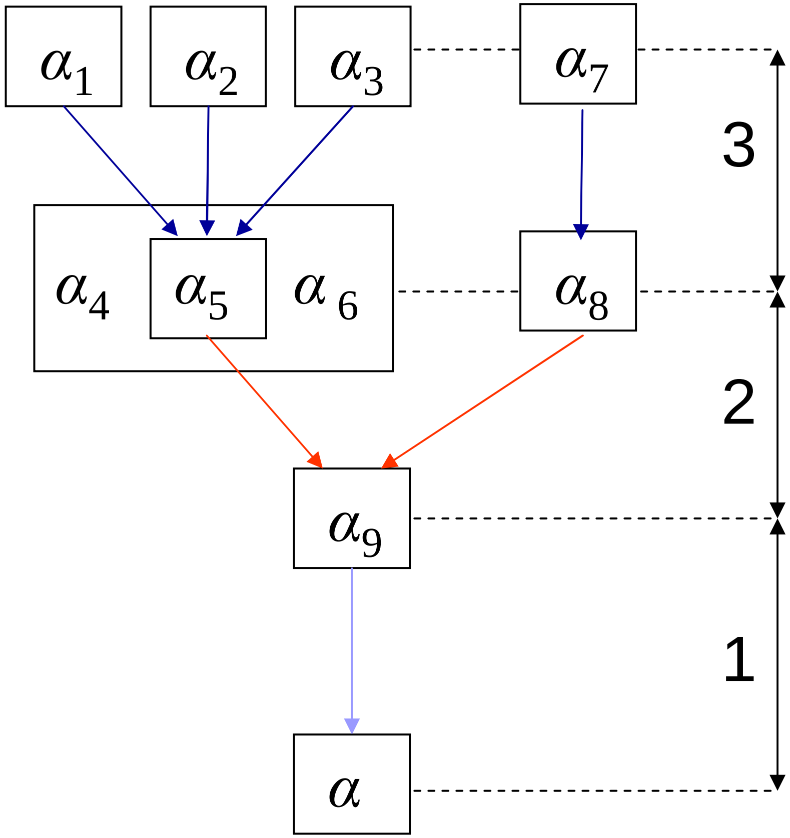
Def. 11

The counterdomain of **A** is the set of all the conclusions of **A**.
i.e. the set of intermediate conclusions ∪ {final colclusion}

Def. 12

The range of **A** is the sum of the domain and counterdomain of **A**.

Example



Domain:

$\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9\}$

Counterdomain:

$\{\alpha, \alpha_5, \alpha_8, \alpha_9\}$

Range:

$\{\alpha, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9\}$

$<\{<\{\alpha_9\}, \alpha>\},$

$\{<\{\alpha_4, \alpha_5, \alpha_6\}, \alpha_9>, <\{\alpha_8\}, \alpha_9>\},$

$\{<\{\alpha_1\}, \alpha_5>, <\{\alpha_2\}, \alpha_5>, <\{\alpha_3\}, \alpha_5>\}, <\{\alpha_7\}, \alpha_8>\}>.$

Def. 13

A sentence δ ***directly supports*** a sentence δ' ***in*** $\mathbf{A}$ iff there exists an atomic argument of $\mathbf{A}$, such that δ' belongs to its domain, and δ belongs to its counterdomain.

Def. 14

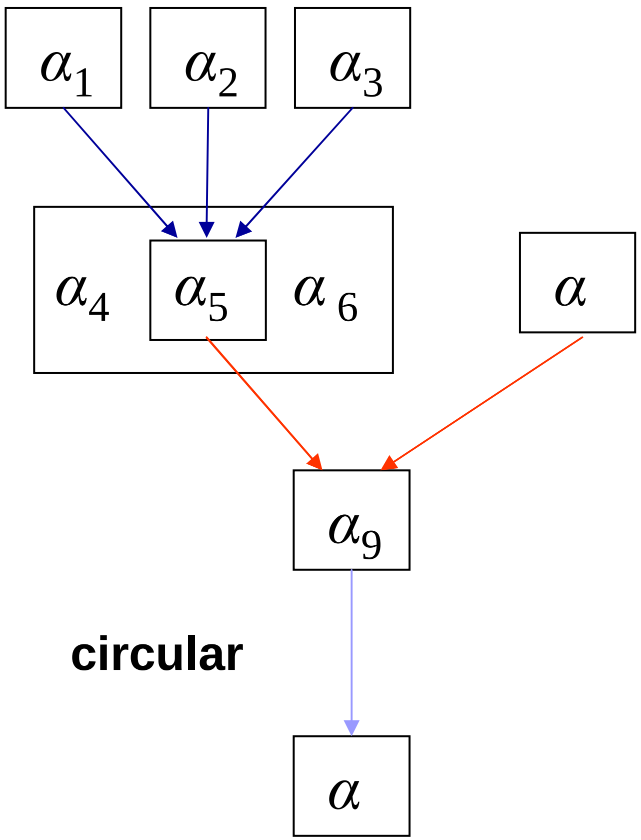
A sentence δ_n ***indirectly supports*** a sentence δ_1 ***in*** $\mathbf{A}$ iff there exists a sequence of sentences $<\delta_1, \delta_2, \ldots \delta_n>$, where $n \geq 3$, such that each of its elements (except for δ_1) directly supports (in $\mathbf{A}$) the preceding element.

Def. 15

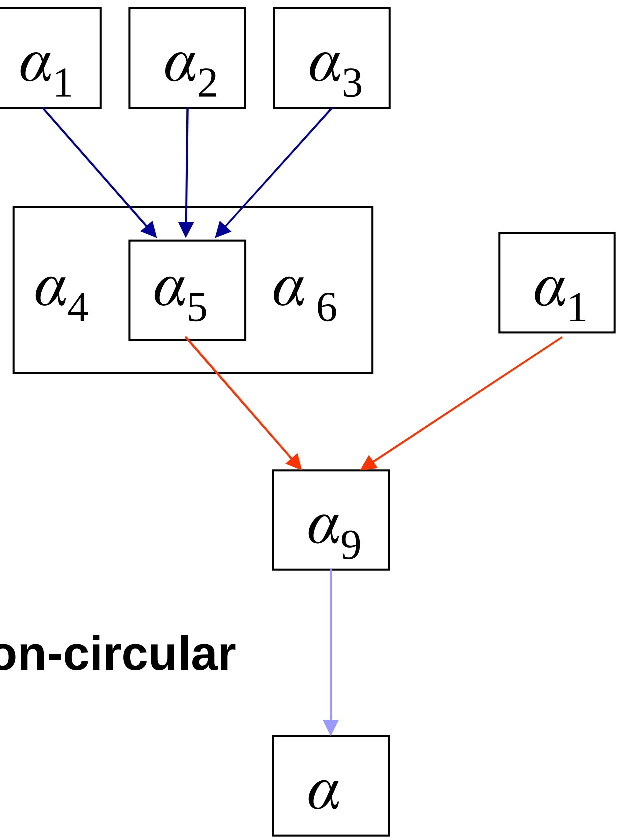
A sentence δ ***supports*** a sentence δ' ***in*** $\mathbf{A}$ iff δ directly or indirectly supports δ' in $\mathbf{A}$.

Def. 16

An argument is ***circular*** iff its range contains a sentence, which supports itself (in this argument).



circular



non-circular

- Assume that $\mathbf{A} = < A_1, A_2, \ldots, A_{n_A}>$ and $\mathbf{B} = <B_1, B_2, \ldots, B_{n_B}>$ are arguments.

Def. 17 ($\mathbf{B} \subseteq \mathbf{A}$)

$\mathbf{B}$ is ***a subargument*** of $\mathbf{A}$ iff the following conditions hold:

(*i*) $n_{\mathbf{B}} \le n_{\mathbf{A}}$;

(*ii*) $\exists k \le n_{\mathbf{A}} - n_{\mathbf{B}} + 1 \; (B_1 \subseteq A_k, \; B_2 \subseteq A_{k+1}, \ldots, \; B_{n_B} \subseteq A_{k+n_B-1})$.

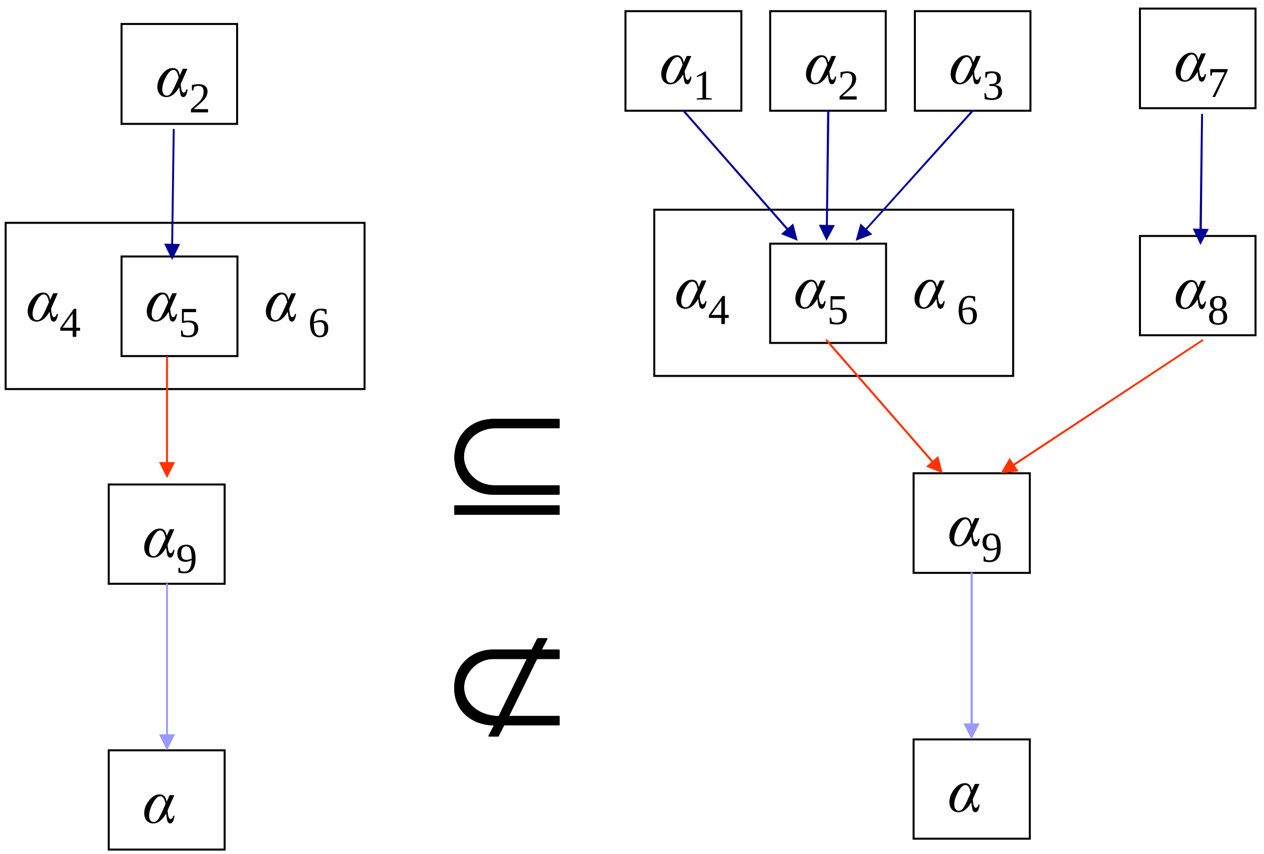
Def. 18 ($\mathbf{B} \subset \mathbf{A}$)

$\mathbf{B}$ is ***an internal subargument*** of $\mathbf{A}$ iff the following conditions hold:

(*i*) $n_{\mathbf{B}} < n_{\mathbf{A}}$;

(*ii*) $\exists k \le n_{\mathbf{A}} - n_{\mathbf{B}} + 1 \; (k > 1$ and
$B_1 \subseteq A_k, \; B_2 \subseteq A_{k+1}, \ldots, \; B_{n_B} \subseteq A_{k+n_B-1})$.

Example



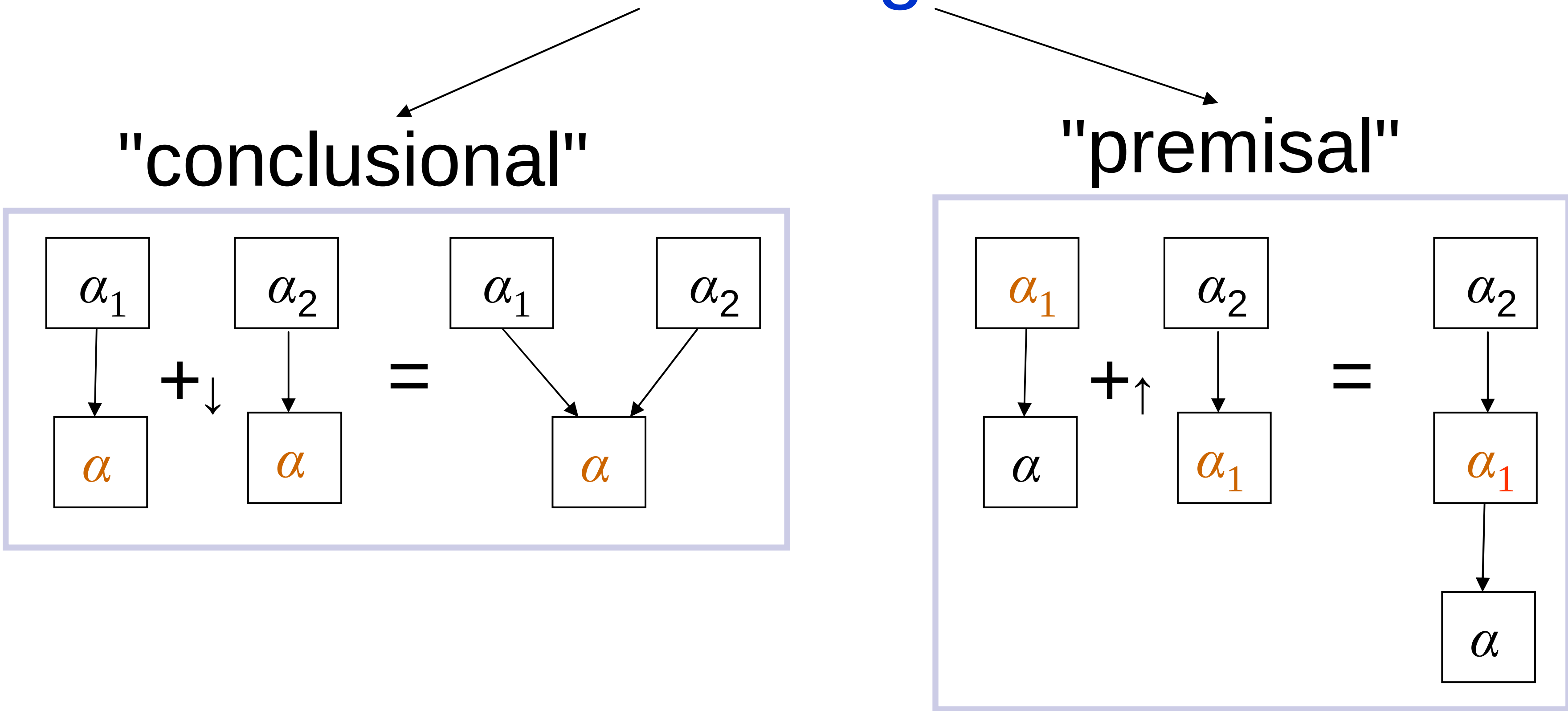
Remark 1: $\mathbf{A} \subseteq \mathbf{A}$, for all $\mathbf{A}$.

Remark 2: "$\mathbf{B} \subset \mathbf{A}$" doesn't mean "$\mathbf{B} \subseteq \mathbf{A}$ and $\mathbf{B} \neq \mathbf{A}$".

2. Operations on arguments.

- Addition.
- Maximal subarguments.
- Subtraction.

Addition of arguments



- Assume that $\mathbf{A} = < A_1, A_2, \ldots, A_{n_A} >$, $\mathbf{B} = < B_1, B_2, \ldots, B_{n_B} >$ and $\mathbf{C} = < C_1, C_2, \ldots, C_{n_C} >$ are arguments.
- Assume that $1 \leq m \leq n_{\mathbf{A}}$.

Def. 19

$$\mathbf{A} +_{\downarrow m} \mathbf{B} = \mathbf{C} \text{ iff}$$

- either the final conclusion of $\mathbf{B}$ is not contained in the counter-domain of A_m and $\mathbf{A} = \mathbf{C}$.
- or the final conclusion of $\mathbf{B}$ is contained in the counterdomain of A_m and the following condisions hold:

(*i*) $n_{\mathbf{C}} = max\ \{n_{\mathbf{A}}\ , m + n_{\mathbf{B}} - 1\}$;

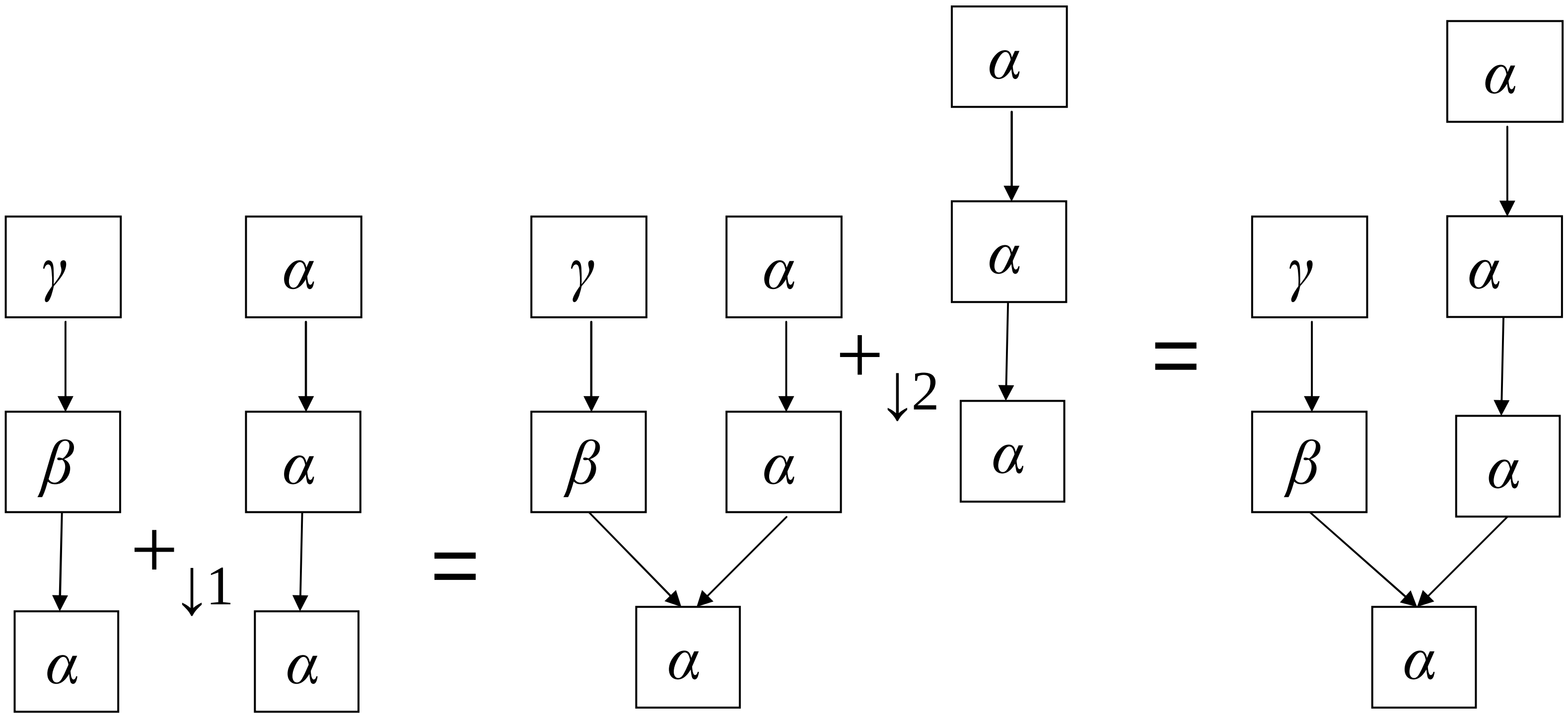
(*ii*) $C_i = A_i$, if $1 \leq i < m$ (for $m \geq 2$) or $i > m + n_{\mathbf{B}}$;

(*iii*) $C_i = A_i \cup B_{i\text{-}m+1}$, if $m \leq i \leq n_{\mathbf{A}}$;

(*iv*) $\mathrm{C}_i = B_{i\text{-}m+1}$, if $n_{\mathbf{A}} < i \leq n_{\mathbf{C}}$.

Def. 20

$$\mathbf{A} +_{\downarrow} \mathbf{B} = (\ldots((\mathbf{A} +_{\downarrow\, n\mathbf{A}} \mathbf{B}) +_{\downarrow\, n\mathbf{A}\text{-}1} \mathbf{B}) +_{\downarrow\, n\mathbf{A}\text{-}2} \ldots) +_{\downarrow 1} \mathbf{B}$$


γ β α
+↓1
α α α
=
γ β α α α
+↓2
α α α
=
γ β α α α α

Def. 21

$$\mathbf{A} +_{\uparrow m} \mathbf{B} = \mathbf{C} \text{ iff}$$

- either the final conclusion of $\mathbf{B}$ is not contained in any element of the domain of A_m and $\mathbf{A} = \mathbf{C}$
- or the final conclusion of $\mathbf{B}$ is contained in some element of the domain of A_m and the following condisions hold:

(*i*) $n_{\mathbf{C}} = max\ \{n_{\mathbf{A}}\ , m + n_{\mathbf{B}}\}$;

(*ii*) $C_i = A_i$, if $1 \leq i \leq m$ (for $m \geq 2$) or $i > m + n_{\mathbf{B}}$;

(*iii*) $C_i = A_i \cup B_{i\text{-}m}$, if $m < i \leq n_{\mathbf{A}}$;

(*iv*) $\mathrm{C}_i = B_{i\text{-}m}$, if $n_{\mathbf{A}} < i \leq n_{\mathbf{C}}$.

Def. 22

$$\mathbf{A} +_{\uparrow} \mathbf{B} = (\ldots((\mathbf{A} +_{\uparrow n\mathbf{A}} \mathbf{B}) +_{\uparrow n\mathbf{A}\text{-}1} \mathbf{B}) +_{\uparrow n\mathbf{A}\text{-}2} \ldots) +_{\uparrow 1} \mathbf{B}$$

Remark 1: Let $m > 1$. Then $\mathbf{A} +_{\downarrow m} \mathbf{B} = \mathbf{A} +_{\uparrow m\text{-}1} \mathbf{B}$ iff

- the final conclusion of $\mathbf{B}$ is contained in the counterdomain of A_m

or

- the final conclusion of $\mathbf{B}$ is not contained in any element of the domain of $A_{m\text{-}1}$.

(*i.e.* the above equation holds iff the final conclusion of $\mathbf{B}$ is not any of the first premises on the level m-1 of $\mathbf{A}$)

Remark 2: The operations of addition are neither commutative nor associative, but if $\mathbf{A}$, $\mathbf{B}$ (and $\mathbf{C}$) have identical final conclusions, then the following equations hold:

$$\mathbf{A} +_{\downarrow 1} \mathbf{B} = \mathbf{B} +_{\downarrow 1} \mathbf{A};$$

$$(\mathbf{A} +_{\downarrow 1} \mathbf{B}) +_{\downarrow 1} \mathbf{C} = \mathbf{A} +_{\downarrow 1} (\mathbf{B} +_{\downarrow 1} \mathbf{C}).$$

Def. 23

$$\mathbf{A} + \mathbf{B} = (\mathbf{A} +_{\uparrow} \mathbf{B}) +_{\downarrow 1} \mathbf{B}$$

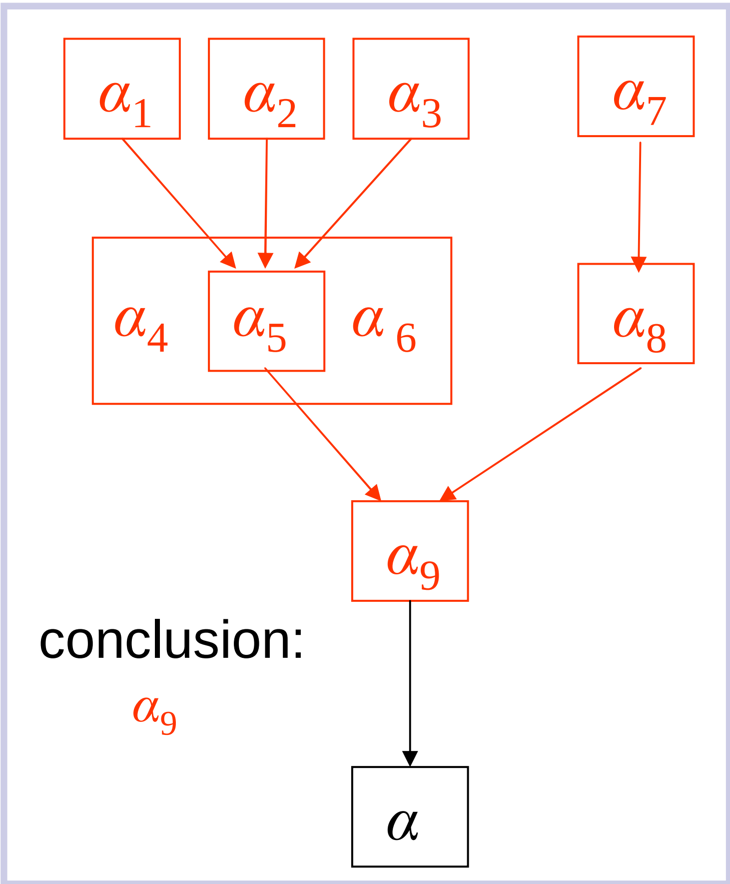
Remark 1: If the final conclusion of $\mathbf{B}$ is not in the range of $\mathbf{A}$, then:

$$\mathbf{A} + \mathbf{B} = \mathbf{A} \,.$$

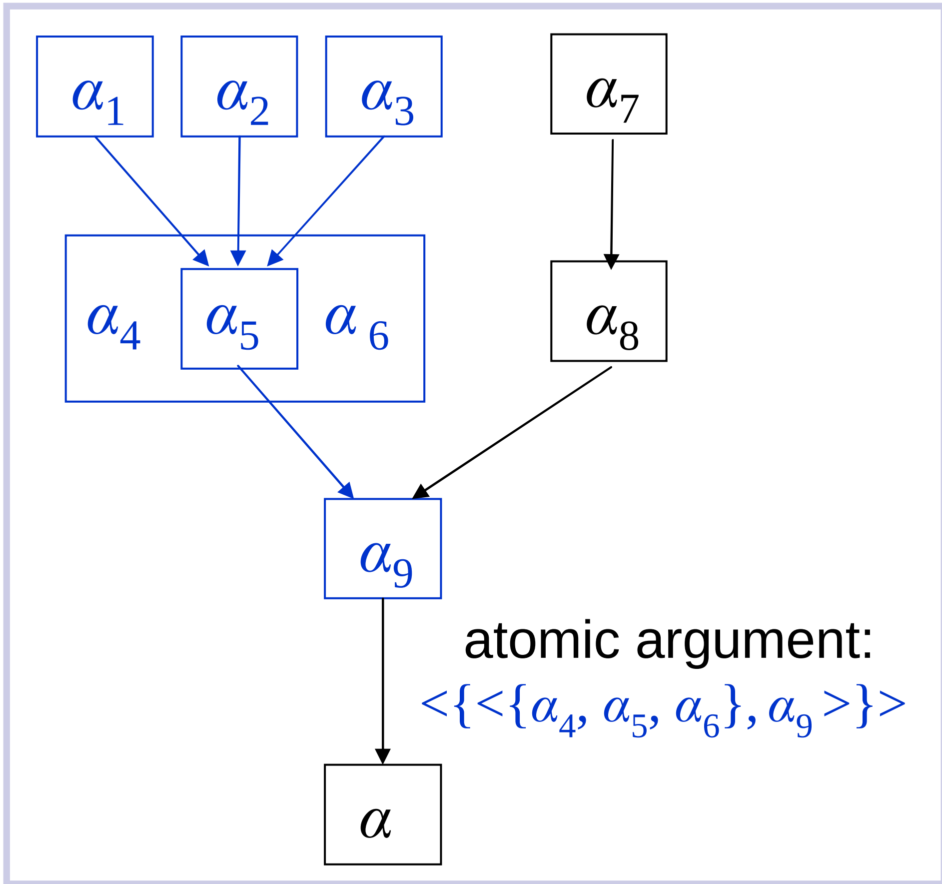
Remark 2: If $\mathbf{A}$ is not circular, then $\mathbf{A} + \mathbf{A} = \mathbf{A}$.

Maximal subarguments

determined by
a conclusion



determined by
an atomic argument



- Assume that $\mathbf{A} = \langle A_1, A_2, \ldots, A_{n_\mathbf{A}} \rangle$ and $\mathbf{B} = \langle B_1 \rangle$ are arguments.
- Assume that $\mathbf{B}$ is an atomic argument in $\mathbf{A}$, where $B_1 \subseteq A_m$ for the level number $m \leq n_\mathbf{A}$.

Def. 24

$\mathbf{C} = max(\mathbf{A}, \mathbf{B}, m)$ iff

$\mathbf{C}$ is the longest (*e.i.* containing the largest number of levels) of the arguments $\mathbf{C^*} = \langle C^*_1, C^*_2, \ldots, C^*_{n_{\mathbf{C}^*}} \rangle$, such that satisfy the following conditions:

(*i*) $n_{\mathbf{C}^*} \leq n_\mathbf{A} - m + 1$;

(*ii*) $C^*_1 = B_1$;

(*iii*) if $n_{\mathbf{C}^*} \geq 2$, then for every $2 \leq i \leq n_{\mathbf{C}^*}$:

$C^*_i = \{\langle P, \delta^* \rangle \in A_{i+m-1}$: δ^* is contained in some element of the domain of $C^*_{i-1}\}$.

- Assume that $\mathbf{A} = \langle A_1, A_2, \ldots, A_{n_\mathbf{A}} \rangle$ is an argument.
- Assume that δ is an element of the counterdomain of A_m for the level number $m \le n_\mathbf{A}$.

Def. 25

$\mathbf{C} = max(\mathbf{A}, \delta, m)$ iff

$\mathbf{C}$ is the longest of the arguments $\mathbf{C^*} = \langle C^*_1, C^*_2, \ldots, C^*_{n_{C^*}} \rangle$, such that satisfy the following conditions:

(*i*) $n_{\mathbf{C^*}} \le n_\mathbf{A} - m + 1$;

(*ii*) $C^*_1 = \{\langle P, \delta^* \rangle \in A_m: \delta^* = \delta\}$;

(*iii*) if $n_{\mathbf{C^*}} \ge 2$, then for every $2 \le i \le n_{\mathbf{C^*}}$:

$C^*_i = \{\langle P, \delta^* \rangle \in A_{i+m-1}$: δ^* is contained in some element of the domain of $C^*_{i-1}\}$.

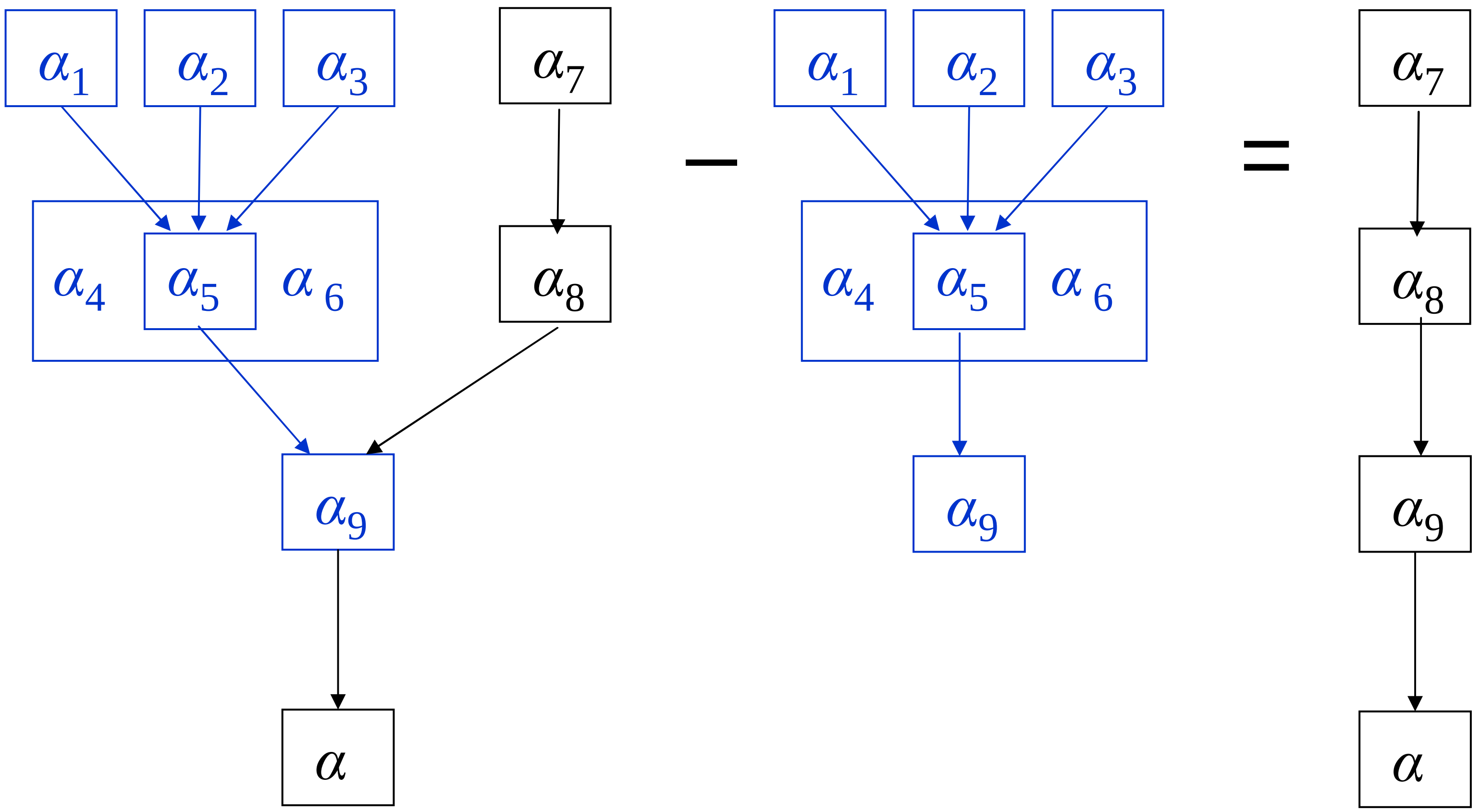
Remark 1: If $\mathbf{B}$ is the final argument of $\mathbf{A}$, then $max(\mathbf{A}, \mathbf{B}, 1) = \mathbf{A}$.

If δ is the final conclusion in $\mathbf{A}$, then $max(\mathbf{A}, \delta, 1) = \mathbf{A}$.

Remark 2: If $\{\mathbf{B}^1, \mathbf{B}^2, \ldots, \mathbf{B}^k\}$ is the set of all atomic arguments of the m-th level of $\mathbf{A}$, which have the same conclusion δ, then:

$$max(\mathbf{A}, \delta, m) = max(\mathbf{A}, \mathbf{B}^1, m) +_{\downarrow 1} max(\mathbf{A}, \mathbf{B}^2, m) +_{\downarrow 1} \ldots +_{\downarrow 1} max(\mathbf{A}, \mathbf{B}^k, m).$$

Subtraction of arguments



- Assume that $\mathbf{A} = <A_1, A_2, \ldots, A_{n_\mathrm{A}}>$ is an argument, and that $\mathbf{B} = <B_1>$ is an atomic (non-final) argument in $\mathbf{A}$ ($B_1 \subseteq A_m$ for $m \leq n_\mathbf{A}$).
- Assume that $\mathbf{C} = <C_1, C_2, \ldots, C_{n_\mathrm{C}}> = max(\mathbf{A}, \mathbf{B}, m)$.

Def. 26

$\mathbf{A} -_m \mathbf{B} = \mathbf{D}$ iff

(*i*) $m - 1 \leq n_\mathbf{D} \leq n_\mathbf{A}$;

(*ii*) if $m \geq 2$, then $D_i = A_i$, for every $i < m$;

(*iii*) if $n_\mathbf{A} = n_\mathbf{C} + m - 1$, then:

- $n_\mathbf{D} = max\{j < n_\mathbf{A}: A_j - C_{j\text{-}m+1} \neq \varnothing\}$;
- $D_i = A_i - C_{i\text{-}m+1}$, for every $m \leq i \leq n_\mathbf{D}$;

(*iv*) if $n_\mathbf{A} > n_\mathbf{C} + m - 1$, then:

- $n_\mathbf{D} = n_\mathbf{A}$;
- $D_i = A_i - C_{i\text{-}m+1}$, for every $m \leq i \leq n_\mathbf{C} + m - 1$;
- $D_i = A_i$, for every $n_\mathbf{C} + m - 1 < i \leq n_\mathbf{D}$.

3. Structural correctness of arguments.

For ***a structurally correct*** argument $\mathbf{A} = <A_1, A_2, \ldots, A_{n_\mathbf{A}}>$ it is necessary that the following conditions hold:

(1) For every argument $\mathbf{B}$:
if $\mathbf{B} \subseteq \mathbf{A}$, then (the counterdomain of $\mathbf{B}$) – (the domain of $\mathbf{B}$) = = {the final conclusion of $\mathbf{B}$}.

(2) (The domain of $\mathbf{A}$) – (the counterdomain of $\mathbf{A}$) = = (the set of all the first premises of $\mathbf{A}$).

(3) For every sentence δ:
if there are $i, j \leq n_\mathbf{A}$, such that the counterdomains of A_i and A_j contain δ, then $max(\mathbf{A}, \delta, i) = max(\mathbf{A}, \delta, j)$.

An open problem: Are these conditions sufficient?

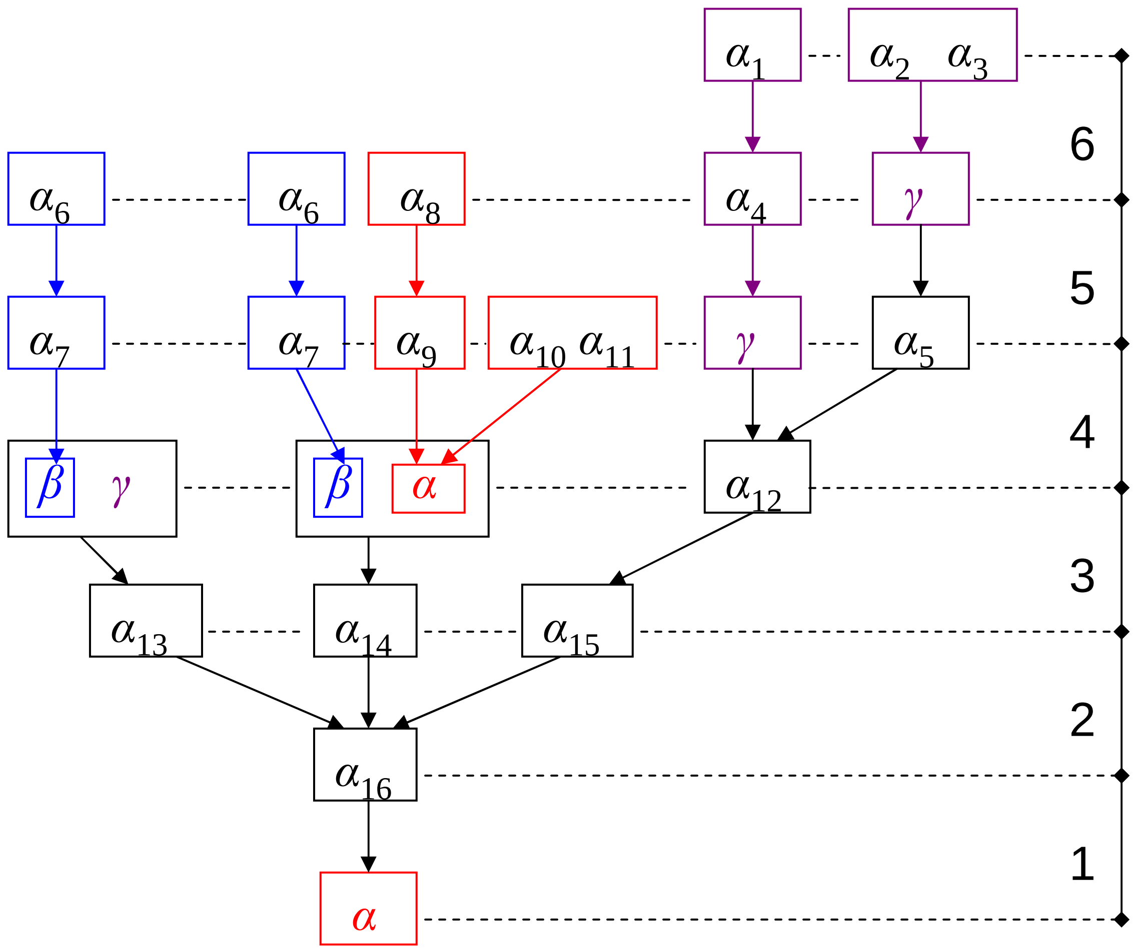
Remark 1: The condition **(1)** doesn't hold iff $\mathbf{A}$ is circular.

Remark 2: The condition **(2)** doesn't hold iff there is a sentence in the domain of $\mathbf{A}$, which is one of the first premises and a conclusion (final or intermediate) at the same time.

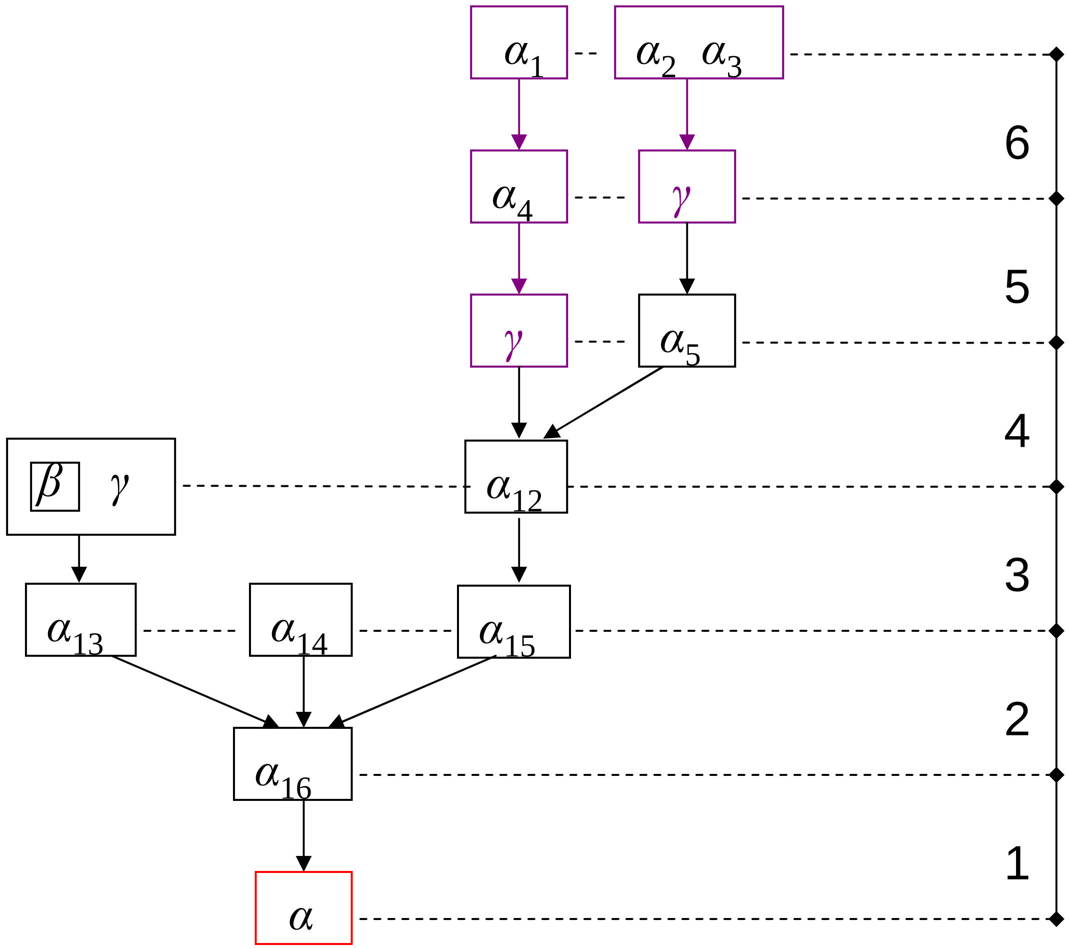
Remark 3: The condition **(3)** doesn't hold iff there is a sentence in the domain of $\mathbf{A}$, which is supported by different subarguments, when it appears on different levels.

Remark 4: If the condition **(1)** doesn't hold, then at least one of the conditions: **(2)** or **(3)** doesn't hold either.
The converse implication is not true.

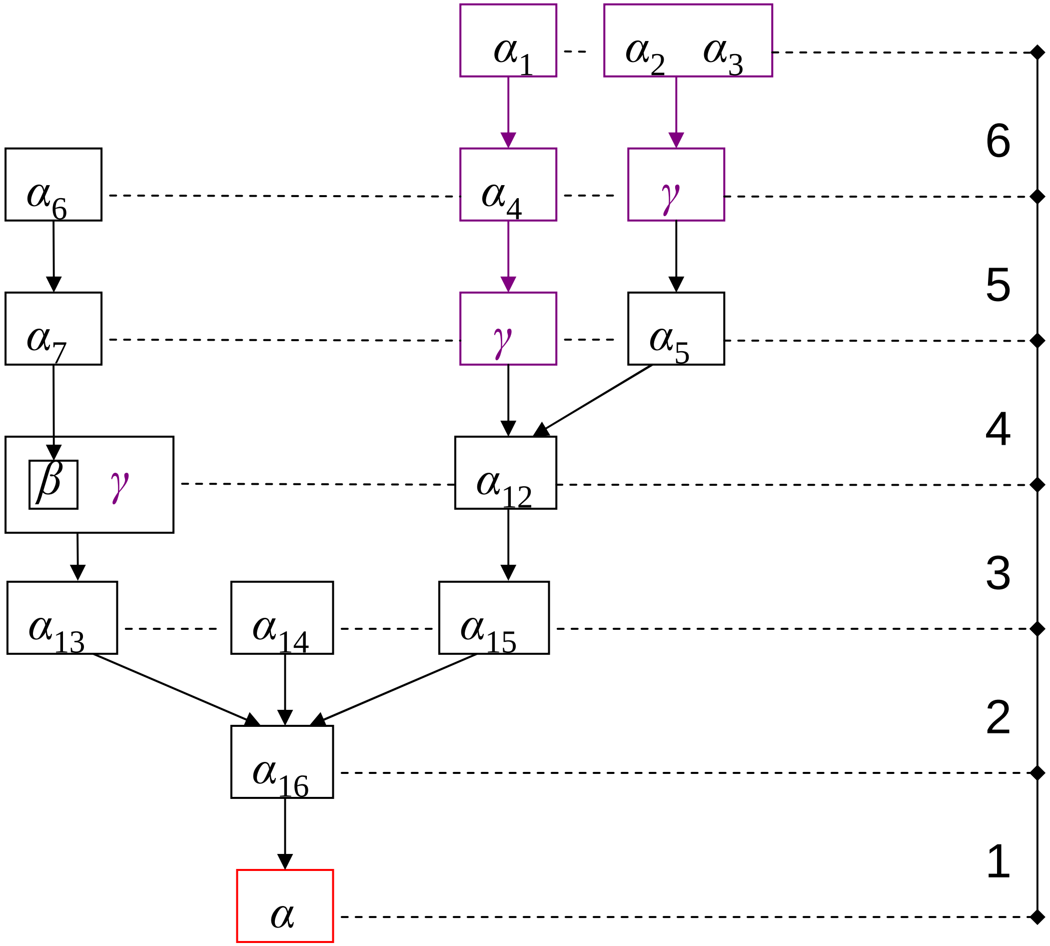
Example



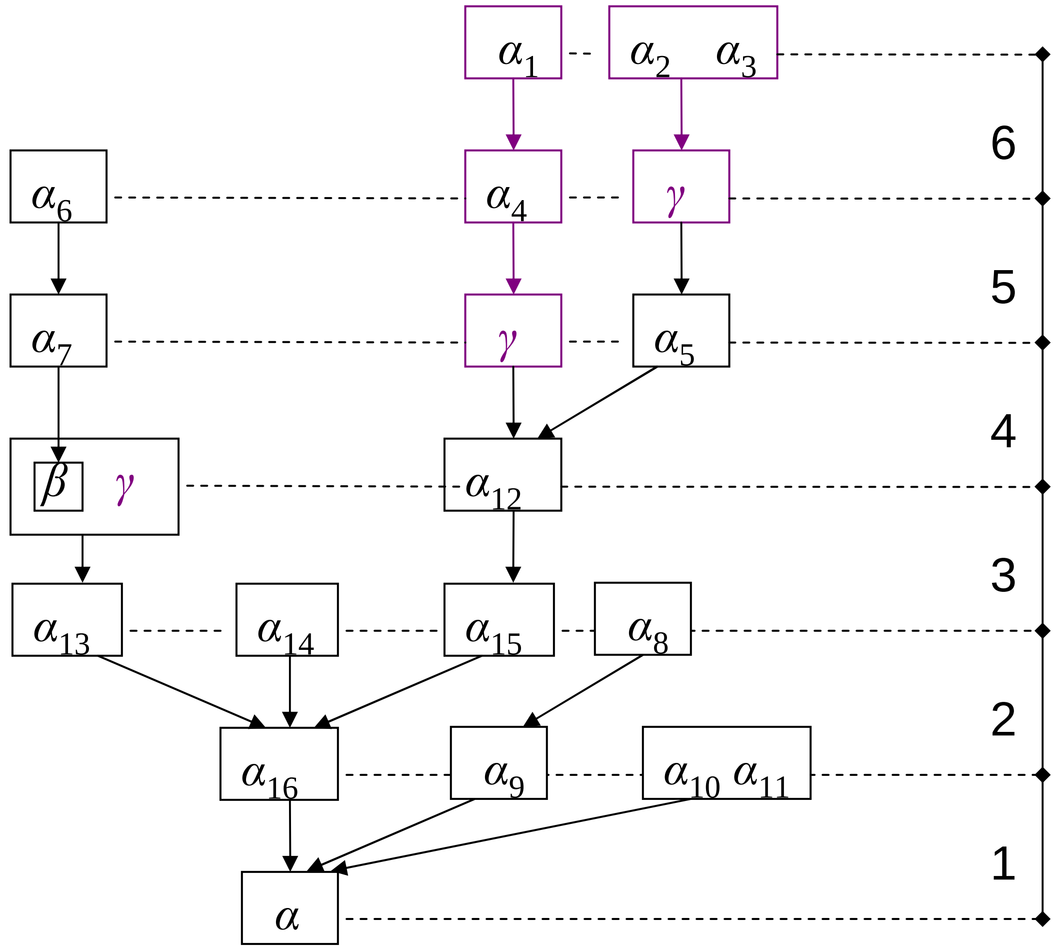
α_1
α_2 α_3
6
α_6
α_6
α_8
α_4
γ
5
α_7
α_7
α_9
α_{10} α_{11}
γ
α_5
4
β γ
β α
α_{12}
3
α_{13}
α_{14}
α_{15}
2
α_{16}
1
α



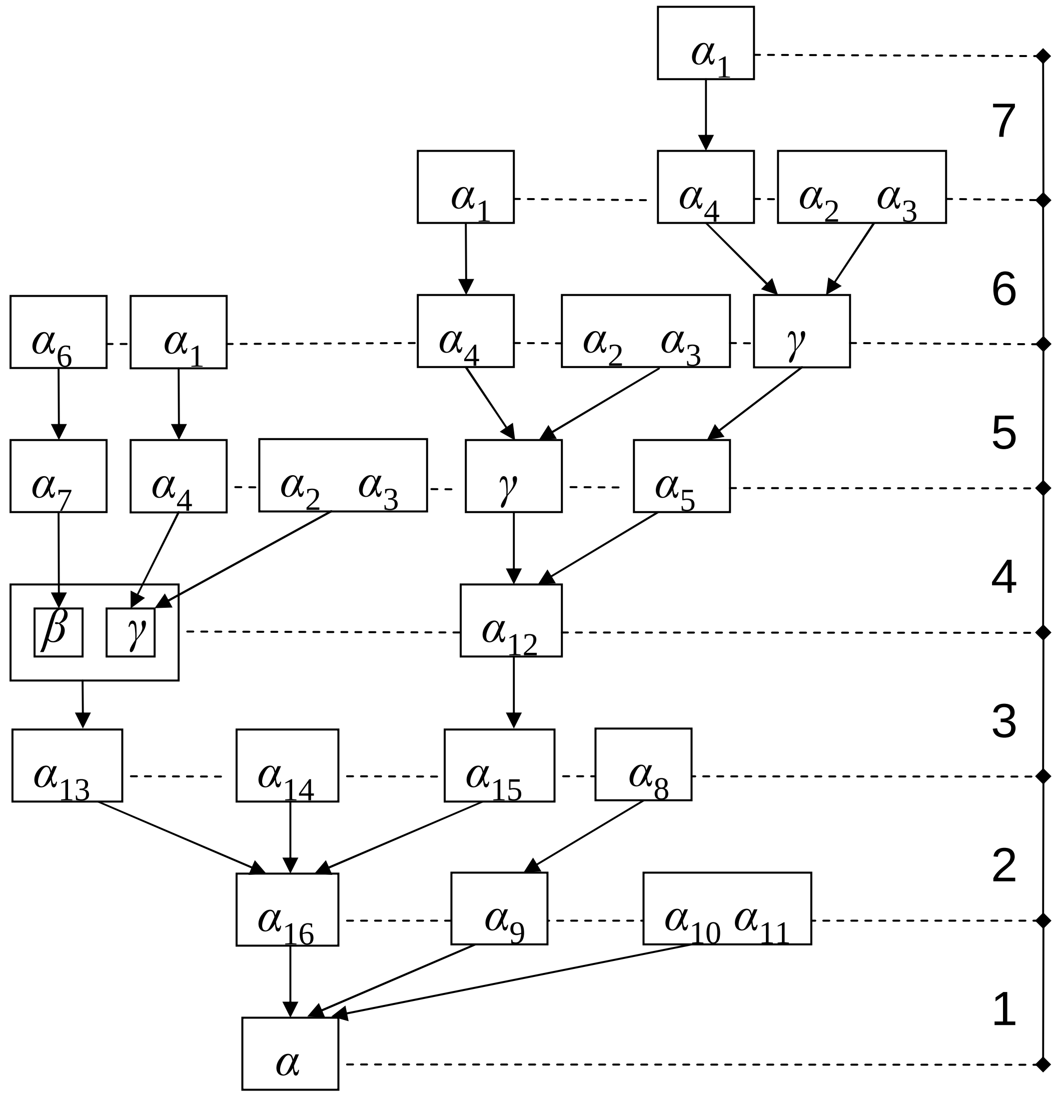
α_1
α_2 α_3
α_4
γ
γ
α_5
β γ
α_{12}
α_{13}
α_{14}
α_{15}
α_{16}
α
6
5
4
3
2
1



α_1
α_2 α_3
α_6
α_4
γ
α_7
γ
α_5
β γ
α_{12}
α_{13}
α_{14}
α_{15}
α_{16}
α
6
5
4
3
2
1



α_1
$\alpha_2 \quad \alpha_3$
α_6
α_4
γ
α_7
γ
α_5
$\beta \quad \gamma$
α_{12}
α_{13}
α_{14}
α_{15}
α_8
α_{16}
α_9
$\alpha_{10} \ \alpha_{11}$
α
6
5
4
3
2
1



α_1
7
α_1 α_4 α_2 α_3
6
α_6 α_1 α_4 α_2 α_3 γ
5
α_7 α_4 α_2 α_3 γ α_5
4
β γ α_{12}
3
α_{13} α_{14} α_{15} α_8
2
α_{16} α_9 α_{10} α_{11}
1
α


- Dziękuję za uwagę.