L – decidability of some invariant systems.

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Outline

- Definition of the invariant system
- System W
- Sobocinski’s system
- Remarks

Invariant systems

- System (X, R) is an invariant system if:
- $Sb(X) = X$, and
- $R \subseteq Struct$,

  where the class of structural rules is:

$$r \in Struct \Leftrightarrow \forall \Pi \subseteq S \ \forall \alpha \in S \ \forall e : At \rightarrow S$$

$$[(\Pi, \alpha) \in r \Rightarrow (h^e(\Pi), h^e(\alpha)) \in r]$$

L-decidable systems

- The system based on a set of rejection rules and a set of rejected axioms is called **L-decidable** if and only if the following conditions hold:

$$(I)\ T \cap T^* = \varnothing$$

$$(II)\ T \cup T^* = S$$

- where T is the set of all theses,
- T* is the set of all rejected formulas, and
- S is the set of all formulas

System W

- The system W is defined by the following matrix:

$$M_w = \left(\left\{0, \frac{1}{2}, 1\right\}, \{1\}, \{c, k, a, \neg\} \right)$$

- where:

$$k(x,y) = \begin{cases} \min(x,y), \ if \ x \neq \frac{1}{2} \ and \ y \neq \frac{1}{2} \\ \frac{1}{2}, \ else \end{cases}$$

$$\neg x = 1 - x$$

$$c(x,y) = \begin{cases} 0, \ if \ x = 1 and (y = 0 \ or \ y = \frac{1}{2}) \\ 1, \ else \end{cases}$$

$$a(x,y) = \begin{cases} \max(x,y), \ if \ x \neq \frac{1}{2} \ and \ y \neq \frac{1}{2} \\ \frac{1}{2}, \ else \end{cases}$$

System W – cont.

- Consider the following functors:

,
$$F_0(p,q) = CCpqCApqKpq$$

,
$$F_{\frac{1}{2}}(p,q) = ACCNpApqApqCpq$$

$$F_1(p,q) = F_0(q,p).$$

System W – cont.

- The following functions correspond to functors in the matrix $M_{W:}$

$$f_0(x,y)=\begin{cases}0, & if \;\; x=0 \;\; and \;\; y=1\\ 1, & if \;\; x\neq 0 \;\; or \;\; y\neq 1\end{cases}$$

$$f_1(x,y)=\begin{cases}0, & if \;\; x=1 \;\; and \;\; y=0\\ 1, & if \;\; x\neq 1 \;\; or \;\; y\neq 0\end{cases}$$

$$f_{\frac{1}{2}}(x,y)=\begin{cases}0, & if \;\; x=1 \;\; and \;\; y=\frac{1}{2}\\ 1, & if \;\; x\neq 1 \;\; or \;\; y\neq \frac{1}{2}\end{cases}$$

Rejected axioms

- The formulas:

$$F_0(p,q),\ F_{\frac{1}{2}}(s,r),\ F_1(t,u)$$

- or generalized disjunctions that can be build from the above formulas.

System W – cont.

- The main theorem:

Let α be an arbitrary formula of the system **W**. If $\alpha \notin T$, then $\alpha \in T^*$.

Sobocinski's system

- The implicational-negational system of Sobocinski is given by the following matrix:

$$\mathrm{M}_S^{c,n} = \left(\{0,1,2,\ldots,k-1\},\{1,2,\ldots,k-1\},\{c,n\}\right)$$

- where k>=3, and for $x, y \in \{0,1,\ldots,k-1\}$

$$c(x,y) = \begin{cases} y, \; if \;\; x \neq y \\ k-1, \; if \;\; x = y \end{cases} \qquad n(x) = \begin{cases} x+1, \; if \;\; x < k-1 \\ 0, \; if \;\; x = k-1 \end{cases}$$

Sobocinski's system –cont.

- Consider the following functors:

$$G_0(p,q) = CpNCqq,$$
$$G_1(p,q) = CpN^2Cqq, \ldots,$$
$$G_{k-2}(p,q) = CpN^{k-1}Cqq$$

- where for l>=1:

$$\begin{cases} N^1\alpha = N\alpha \\ N^{l+1}\alpha = NN^l\alpha \end{cases}$$

Sobocinski's system – cont.

- The following functions correspond to functors in the matrix $M_S^{c,n}$:
- $$g_0(x,y)=\begin{cases}0, & if \;\; x\neq 0\\ k-1, & if \;\; x=0\end{cases}$$
- $$g_1(x,y)=\begin{cases}1, & if \;\; x\neq 1\\ k-1, & if \;\; x=1\end{cases}$$
- ……………………
- $$g_{k-2}(x,y)=\begin{cases}k-2, & if \;\; x\neq k-2\\ k-1, & if \;\; x=k-2\end{cases}$$

Sobocinski's system – cont.

- The negations of the functions defined on the previous slides:
- $$n(g_0(x,y)) = \begin{cases} 1, & if \;\; x \neq 0 \\ 0, & if \;\; x = 0 \end{cases}$$
- $$n(g_1(x,y)) = \begin{cases} 2, & if \;\; x \neq 1 \\ 0, & if \;\; x = 1 \end{cases}$$
- ……………………
- $$n(g_{k-2}(x,y)) = \begin{cases} k-1, & if \;\; x \neq k-2 \\ 0, & if \;\; x = k-2 \end{cases}$$

Sobocinski's system – cont.

- Now we can define the following functors:
  - $$F_{k-1}(p,q) = CNG_0(q,p)CNG_1(q,p)\ldots$$
    $$\ldots CNG_{k-3}(q,p)CNG_{k-2}(q,p)NG_{k-2}(p,q)$$
  - $$F_{k-2}(p,q) = CF_{k-1}(p,q)CNG_0(q,p)CNG_1(q,p)\ldots$$
    $$\ldots CNG_{k-3}(q,p)NG_{k-2}(p,q)\ldots\ldots\ldots$$
  - $$F_1(p,q) = CF_{k-1}(p,q)CF_{k-2}(p,q)\ldots$$
    $$\ldots.CF_2(p,q)CNG_0(q,p)NG_{k-2}(p,q),$$
  - $$F_0(p,q) = CF_{k-1}(p,q)CF_{k-2}(p,q)\ldots CF_1(p,q)NG_{k-2}(p,q)$$

Sobocinski's system – cont.

- The following functions correspond to functors in the matrix $M_S^{c,n}$:

$$f_l(x,y)=\begin{cases} 0, \; if \;\; x=k-2 \; and \;\; y=l \\ k-1, \; if \;\; x\neq k-2 \; or \;\; y\neq l \end{cases}$$

Sobocinski's system – cont.

- Now consider the following functor:

$$A_S(p,q) = CN^2CppCCqpNG_0(p,q).$$

- The following function corresponds to the functor in the matrix $M_S^{c,n}$:

$$a_s(x,y) = \max\{x,y\} \text{ for x, y} \in \{0, k-1\}$$

Rejected axioms

- The formulas:
- $F_i(p,q)$ or
- $A_S(A_S(F_i(r,p),F_j(q,s)),\ldots),F_t(u,v))$

Sobocinski's System – cont.

- The main theorem:

Let α be an arbitrary formula of the Sobocinski system.

If $\alpha \notin T$, then $\alpha \in T^*$.