## SECTION 2: LOGIC AND COMPUTATION

## Questions, Recursion, and Incompleteness

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The main result is a theorem showing that some natural, reasonable assumptions concerning questions and answers give rise to certain incompleteness phenomena of question-answers systems. These assumptions are formulated in terms of Recursion Theory and correspond to decidability or positive decidability of: the set of all questions of a given language as well as to the sets of direct answers to particular questions of this language. We do not need any assumptions concerning the syntax except a very natural demand that the set of all sentences is recursive.

Our results strengthen Harrah's incompleteness theorem in an essential way. Among others, we show that (granted the natural assumptions mentioned above) one can find, in a recursive way, an infinite family of recursive sets of sentences being not equal to the sets of direct answers to any question. Thus, metaphorically speaking, there exist sets of answers which are not accessible to any reasonable question system. Moreover, we give natural sufficient conditions for the existence of problems whose sets of solutions are not even positively decidable.

We also draw some epistemic consequences of the main result. Finally, the purely mathematical content of our theorems seems to have applications also beyond question-answer systems.