Semiotyka logiczna

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Dodatek 15.1.

Triada hipotaktyczna: część 1

Niniejsza prezentacja zawiera tekst artykułu ***An axiom system for hypotaxis***, opublikowanego w 1981 roku w serii preprintów *Working Papers of the Institute of Linguistics, Adam Mickiewicz University, Poznań*.

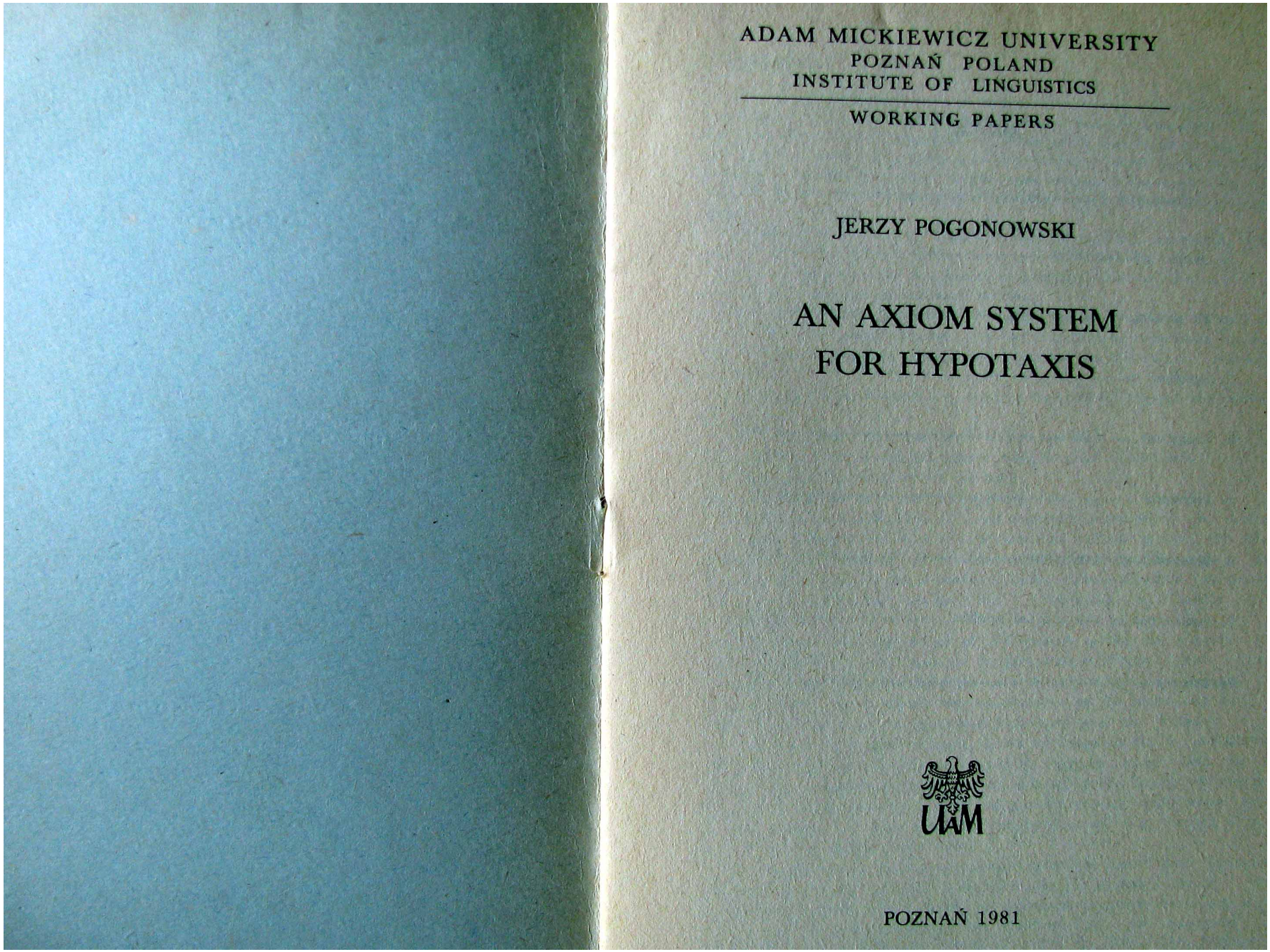
W artykule podaje się m.in. system aksjomatów dla determinacyjnej teorii języka, sformułowanej przez Profesora Jerzego Bańczerowskiego.

Triada hipotaktyczna: część 1

System ten był później badany m.in. w następujących pracach piszącego te słowa:

- *Grundideen der kombinatorischen Semantik.* SAIS Arbeitsberichte, Christian-Albrechts-Universität, Kiel 1985, 133-152.
  [Zobacz Dodatek 16.]
- *Hiponimia.* Wydawnictwo Naukowe UAM, Poznań 1991.
  [Zobacz Dodatek 17.]
- *Combinatory semantics.* Wydawnictwo Naukowe UAM, Poznań 1993.

Ze względu na dużą objętość pliku, został on podzielony na dwa fragmenty. To jest pierwszy fragment.



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WORKING PAPERS

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AN AXIOM SYSTEM FOR HYPOTAXIS

POZNAŃ 1981

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0. Introduction

The goal of the present work is to provide a formal characterization of the relation of hypotaxis /syntactic subordination/. This paper is a draft version of a more comprehensive study which aims to be a step toward a formal theory of hypotaxis. Below, we limit ourselves to the presentation of the proposed axiom system, some simple consequences of the axioms and a few remarks concerning possible extensions of the initial system.

The method accepted in this work is that of informal axiomatics. This means that :

- there are some concepts considered as primitive /non-definable/ ; it is assumed that they are intuitively obvious
- all considered concepts are either primitive or defined in a formal way
- primitive and defined concepts are characterized by postulates /axioms/, i.e. statements accepted without proof, on the basis of their self-evidence only
- all considered statements are either postulates or logical consequences of postulates.

There are three primitive terms of our system :

dim denoting the set of all proper morphological dimensions
hpn denoting the relation of hyponymy
hph denoting the relation of homophony.

The intuitive explanation of these concepts is presented in sections 2-4. These concepts, as well as some concepts defined by them, are characterized by a system of seven axioms. The axioms are split into four groups with respect to their content. Thus, axiom 1 concerns morphology, axioms 2,3,4 - hyponymy, axioms 5 and 6 - homophony, and, finally, axiom 7 - hypotaxis.

A few words about the origin of the present system are in order. The first version of it was presented in part C of POGONOWSKI 1982. There, the above three primitive concepts were characterized by a system of sixteen axioms. Those axioms are consequences of the present system.

The formal approach to hypotaxis presented here is related to J. Bańczerowski's determinational theory of language /cf. BAŃCZEROWSKI 1980/. In fact, determinational theory of language can be based on our axiom system. Several shortcomings of Bańczerowski's theory are discussed in POGONOWSKI 1982. We think that the present system avoids those shortcomings. However, it also has some limitations, as it is thought of as a part of syntax and semantics only.

There are several links between our system and some other linguistic theories concerning syntactic subordination. For example, there exist connections between formal theory of hypotaxis presented here and structural syntax of Lucien Tesnière /cf. TESNIÈRE 1959/, the concept of determination as presented in REFORMATSKIJ 1967 /p. 325/, the concept of

configuration investigated in algebraic linguistics /cf. MARCUS 1970, ch. 6/ and D.G. Hays' dependency grammars /cf. HAYS 1964/.

1. Formal preliminaries

In this section we present all the formal concepts necessary for the understanding of the text. These concepts belong to the calculus of sets and relations and to the system of extended mereology. We assume that the reader has at least rudimentary familiarity with the elementary calculus of sets and relations. The system of extended mereology is presented in TARSKI 1937 /cf. also the exposition of this system in BATÓG 1967/. The notation used here is standard. Thus, we limit ourselves to the presentation of those formal concepts occurring in the formulation of the axioms.

The symbols $\in, \notin, \subset, \emptyset$ stand for membership, the negation of membership, inclusion and the empty set, respectively. The set of all x which have the property W is denoted by $\{x : x \text{ has the property } W\}$. The logical connectives of negation, conjunction, disjunction, implication and equivalence are denoted by $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$, respectively. The symbols $\bigvee_x$ and $\bigwedge_x$ denote the existential and the universal quantifier.

If X and Y are sets, then $X \cup Y$, $X \cap Y$, $X - Y$ and $X \times Y$ denote the union, intersection, difference and Cartesian product of X and Y, respectively. The ordered pair consisting of x and y is denoted by (x,y). If X is a set, then $\mathcal{P}(X)$ denotes the powerset of X and X^n denotes the n-th Cartesian power of X. The cardinality of a set X is denoted by card(X). Given a family $\mathcal{A}$ of sets, the union and the intersection of $\mathcal{A}$ are denoted by $\bigcup\mathcal{A}$ and $\bigcap\mathcal{A}$, respectively. The fact that f is a function from X into Y is denoted by $f : X \rightarrow Y$. If X is a set and R is an equivalence relation on X, then the partition of X into R-equivalence classes is denoted by X/R.

We say that a set X is :

<u>hereditary finite</u>, if X is finite and all elements of X are finite

<u>hereditary non-empty</u>, if X is non-empty and all elements of X are non-empty

<u>hereditary disjoint</u>, if for all distinct $x,y \in X$, x and y are disjoint and for any $z \in X$ and $u,v \in z$, if $u \neq v$, then u and v are disjoint.

A set is called a <u>stratified hierarchy</u>, if it is hereditary finite, hereditary non-empty and hereditary disjoint. The class of all stratified hierarchies of cardinality at least n is denoted by SH(n).

Given a binary relation $R \subset X \times Y$ and $x \in X$, $y \in Y$, $A \subset X$, $B \subset Y$ let us denote :

$R^{\frown}x = \{z \in Y : x\,R\,z\}$ $\qquad$ $R^{\smile}y = \{z \in X : z\,R\,y\}$

$R[A] = \bigcup\{R^{\frown}x : x \in A\}$ $\qquad$ $R\langle B\rangle = \bigcup\{R^{\smile}y : y \in B\}$

$\mathrm{dom}(R) = R\langle Y\rangle$ /<u>domain</u> of R/ $\qquad$ $\mathrm{rng}(R) = R[X]$ /<u>range</u> of R/

$\mathrm{fld}(R) = \mathrm{dom}(R) \cup \mathrm{rng}(R)$ /<u>field</u> of R/

$\mathrm{conv}(R) = \{(v,u) : (v,u) \in Y \times X \wedge u\,R\,v\}$ /<u>converse</u> of R/.

A binary relation is a <u>partial ordering</u>, if it is transitive and reflexive in its field. A partial ordering R is <u>finitary upwards</u>, if there is no infinite chain $x_0, x_1, x_2, \ldots$ such that $x_i\,R\,x_{i+1}$ for $i \geqslant 0$ and $x_i \neq x_j$ for $i \neq j$. The class of all finitary upwards partial orderings is denoted by FU. By an <u>equivalence induced by a partial ordering</u> R we mean the relation $\mathrm{eqv}(R) = R \cap \mathrm{conv}(R)$.

An equivalence R on X is said to be of a <u>finite index over</u> Y, where $Y \subset X$, if $Y/(R \cap Y^2)$ is finite /i.e. if R restricted to Y has a finite number of equivalence classes/. The class of all equivalences on X with a finite index over Y is denoted by FI(X,Y).

If R is an equivalence on X and $Y \subset X$, then we will often consider R as an equivalence on Y. In such a case, we will write simply R instead of $R \cap Y^2$ /and, similarly, Y/R instead of $Y/(R \cap Y^2)$/.

A relation $R \subset X^2$ is <u>compatible</u> with an equivalence E on X, if $R \cap E = \emptyset$ and for all $x,y,u,v \in X$, if $x\,E\,u$, $y\,E\,v$ and $x\,R\,y$, then $v\,R\,u$ does not hold.

By the <u>reduct</u> of a binary transitive relation R we mean the relation $\{(x,y) : x\,R\,y \wedge \neg\bigvee_z (z \neq x \wedge z \neq y \wedge x\,R\,z \wedge z\,R\,y)\}$.

A system (X,R,x) is called a <u>tree</u>, if R is a transitive and reflexive relation on X, x is the R-greatest element of X and for any $y,u,v \in X$, if $y \neq x$, $y\,R\,u$ and $y\,R\,v$, then either $u\,R\,v$ or $v\,R\,u$. If (X,R,x) is a tree, then R is called a <u>tree ordering</u> of X and x is the <u>root</u> of this tree.

(X,R,x) is a <u>subtree</u> of a tree (Y,Q,y), if (X,R,x) is a tree, $X \subset Y$ and $R \subset Q$.

A system (X,R,x) is a <u>reduced tree</u>, if R is the reduct of some tree ordering of X and x is the root of the corresponding tree.

A system $(\mathfrak{X},R,E)$ is a <u>uniform forest</u>, if :

1. $\mathfrak{X} = \{(X_i,x_i) : i \in I\}$, I an arbitrary index set
2. E is an equivalence on $\bigcup\{X_i : i \in I\}$
3. $(X_i, R \cap X_i^2, x_i)$ is a reduced tree for all $i \in I$
4. R is compatible with E.

The class of all uniform forests is denoted by UFO.

We are not going to present systematically the system of extended mereology here - for the exposition of the axiomatics of this system cf. the works of Tarski and Batóg quoted above. Below, we recall the main concepts of mereology and introduce a few new technical notions which are useful for the presentation of our axiom system.

The system of extended mereology has two primitive concepts : P /denoting the relation of being a part/ and T /denoting the relation of precedence in time/. Hence the expressions $x\,P\,y$ and $x\,T\,y$ should be read "a thing x is a part of a thing y" and "a whole thing x precedes in time a whole thing y or the last time slice of x coincides with the first time slice of y", respectively. The definitions of some further mereological concepts are listed below :

$y = S(X) \longleftrightarrow X \subset P^{\vee}y \wedge \bigwedge_{z} (z\ P\ y \longrightarrow P\langle X\rangle \cap P^{\vee}z \neq \emptyset)$

/mereological union of the set X/

$x \dot{\cup} y = S(\{x,y\})$ /mereological union of x and y/

$x \dot{\cap} y = S(P^{\vee}x \cap P^{\vee}y)$ /mereological intersection/

$x \dot{-} y = x \dot{\cap} S(\{z : P^{\vee}z \cap P^{\vee}y = \emptyset\})$ /mereological difference/

$mo = \{x : x\ T\ x\}$ /momentary things/

$pn = \{x : P^{\vee}x = \{x\}\}$ /points/

$x\ C\ y \longleftrightarrow x\ T\ y \wedge y\ T\ x$

$ms = mo \cap \{x : C^{\vee}x \subset P^{\vee}x\}$ /momentary world-sections/

$x\ T_c\ y \longleftrightarrow \bigwedge_{u} \bigwedge_{v} (u\ P\ x \wedge v\ P\ y \longrightarrow \neg v\ T\ u)$ /complete precedence in time/

$x\ T_e\ y \longleftrightarrow x\ T_c\ y \vee x = y$

$x\ T_i\ y \longleftrightarrow x\ T_c\ y \wedge \neg \bigvee_{z} (x\ T_c\ z \wedge z\ T_c\ y)$ /immediate precedence in time/

$ln = \{x : \bigwedge_{y} (y \in ms \wedge \neg y\ T_c\ x \wedge \neg x\ T_c\ y \longrightarrow x \dot{\cap} y \in pn)\}$ /linear objects/

$eln = P\langle ln\rangle$.

A partial ordering $R \subset X^2$ is called <u>mereological</u>, if for all distinct $x,y \in X$: if $x\ P\ y$, then not $x\ R\ y$. The class of all mereological partial orderings in X is denoted by MPO(X).

A binary relation R on X is called a <u>mereological congruence</u>, if :

1. R is an equivalence on X
2. for all $x,y,u,v \in X$, if $x \dot{\cup} y, x \dot{-} y, u \dot{\cup} v, u \dot{-} v \in X$ and $x\ R\ u$, $y\ R\ v$, then also $x \dot{\cup} y\ R\ u \dot{\cup} v$ and $x \dot{-} y\ R\ u \dot{-} v$
3. for all $x,y,u \in X$, if $x\ R\ y$ and $u\ P\ x$, then there is $v \in X$ such that $v\ P\ y$ and $u\ R\ v$
4. for all $x,y,u,v \in X \cap ln$, if $x\ R\ u$, $y\ R\ v$, $x\ T\ y$ and $x \dot{\cup} y$, $u \dot{\cup} v \in eln$, then $u\ T\ v$.

The class of all mereological congruences on X is denoted by MC(X).

A system $(X_1,\ldots,X_k)$, $k>1$, is a <u>hierarchical family of partitions</u>, if

1. $S(X_i) = S(X_j)$ for $1 \leqslant i,j \leqslant k$
2. for all i such that $1 \leqslant i \leqslant k$ and for all distinct $x,y \in X_i$, $P^{\vee}x \cap P^{\vee}y = \emptyset$
3. for all i such that $1 < i \leqslant k$ and for all $x \in X_i$ there exists $y \in X_{i-1}$ such that $x\ P\ y$.

The class of all hierarchical families of partitions is denoted by HFP.

A set X is a <u>mereological partition</u> of a thing x, if $S(X) = x$ and for any $y,z \in X$, if $y \neq z$, then $P^{\vee}y \cap P^{\vee}z = \emptyset$.

2. Morphology

This section is a brief exposition of abstract morphology. The concept of a proper morphological dimension is characterized by axiom 1. The set dim of all <u>proper morphological dimensions</u> is used in the construction of further linguistic concepts, viz. those of proper morphemes, morphemes and actual lexes. We formulate the axiom and the necessary definitions, as well as a few simple propositions, first ; then

all the investigated concepts will be given the explicit linguistic interpretation.

<u>AXIOM 1</u>. $dim \in SH(2)$

2.1. <u>Definition</u>. $mph = \bigcup dim$. Elements of the set mph are called <u>proper morphemes</u>.

2.2. <u>Definition</u>. $Lex = \bigcup mph$. Elements of the set Lex are called <u>actual lexes</u>.

2.3. <u>Definition</u>. The function $idf : dim \longrightarrow \mathcal{P}(Lex)$ is defined by $idf(\mathfrak{A}) = Lex - \bigcup \mathfrak{A}$ for any $\mathfrak{A} \in dim$. The set $idf(\mathfrak{A})$ is called the <u>indefinite morpheme</u> of $\mathfrak{A}$.

2.4. <u>Definition</u>. $Mph = mph \cup idf[dim]$. Elements of the set Mph are called <u>morphemes</u>.

2.5. <u>Definition</u>. $Dim = \{\mathfrak{A} \cup \{idf(\mathfrak{A})\} : \mathfrak{A} \in dim\}$. Elements of the set Dim are called <u>morphological dimensions</u>.

The following simple propositions follow immediately from axiom 1.

2.6. <u>Proposition</u>. The set dim of all proper morphological dimensions, as well as any particular proper morphological dimension, are finite and non-empty.

2.7. <u>Proposition</u>. Any two distinct proper morphological dimensions are disjoint.

2.8. <u>Proposition</u>. Any two distinct proper morphemes belonging to the same proper morphological dimension are disjoint.

2.9. <u>Proposition</u>. For any proper morphological dimension $\mathfrak{A}$, if $\mathfrak{A}$ contains at least two elements, then all proper morphemes belonging to $\mathfrak{A}$ are non-empty.

2.10. <u>Proposition</u>. The sets of all proper morphemes and of all actual lexes are both non-empty.

2.11. <u>Proposition</u>. Every actual lex belongs to exactly one morpheme in each morphological dimension.

2.12. <u>Proposition</u>. dim is a partition of mph.

2.13. <u>Proposition</u>. For any proper morphological dimension $\mathfrak{A}$, if $idf(\mathfrak{A}) = \emptyset$, then $\mathfrak{A}$ is a partition of Lex.

2.14. <u>Proposition</u>. For any morphological dimension $\mathfrak{A} \in Dim$, $\mathfrak{A}$ is a partition of Lex if and only if $\mathfrak{A} = \mathfrak{B} \cup \{idf(\mathfrak{B})\}$ and $idf(\mathfrak{B}) \neq \emptyset$ for some $\mathfrak{B} \in dim$.

2.15. <u>Proposition</u>. The intersection of all indefinite morphemes is empty.

Now let us present the linguistic interpretation of the concepts introduced above. Morphology is understood here in an extensional way. Morphological constructions presented in axiom 1 and definitions 2.1. - 2.5. provide for a hierarchical categorization of the lexical universe with respect to several kinds of morphological information.

At the very beginning, it should be stressed that our system is applicable to an arbitrary natural language. The set dim of all proper morphological dimensions corresponds to all kinds /sorts/ of information

contained in the lexical units of language. In any natural language one can meet two groups of kinds of information : lexical and grammatical. Both these groups are further diversified. The set dim contains all kinds of grammatical as well as lexical information present in the lexical units of the investigated language. Hence, as elements of the set dim we may list such grammatical kinds of information as :

G1 NUMERUS
G2 GENUS
G3 TEMPUS
G4 PERSONA
G5 MODUS
G6 CASUS
G7 GRADATIO QUALITATIS
G8 DIATHESIS
G9 ASPECTUS
etc.

One has to remember that every language has a specific set of kinds of grammatical information. For instance, in Hungarian and Finnish there is no grammatical gender, in Kwakiutl NUMERUS is not expressed grammatically, etc.

Lexical information contained in a given lexical unit of language can be considered in two ways : either as a whole sui generis or as a combination of some lexical primitives /those primitives being simultaneously divided into sorts/. We accept the second approach here. Consequently, the set dim is intended to contain such sorts of lexical information as :

L1 DENOTING TECHNIQUE
L2 DENOTING RANGE
L3 ONTOLOGICAL CATEGORY
L4 COGNITIVE ACCESSIBILITY
L5 MOOD OF ORIGIN
L6 ASPECT OF COLLECTIVITY
L7 AGE
L8 KINSHIP
L9 SEXUALITY /NATURAL GENDER/
L10 SHAPE
L11 SIZE
L12 COLOUR
L13 MOTION
L14 VELOCITY
L15 DEGREE OF AFFECTIVITY
L16 EVALUATION
etc.

As the reader probably observed, the elements L1 - L16 listed above are similar to the abstract entities considered in componential analysis /cf. LYONS 1977, section 9.9/ or in the metalanguage of lexical description /cf. APRESJAN 1980, ch. 2/. They will be discussed once more in this work, in section 7 /cf. also BAŃCZEROWSKI-POGONOWSKI-ZGÓŁKA 1982, ch. 22A/.

Any proper morphological dimension is an abstract entity. Namely, any proper morphological dimension is a set of proper morphemes. Proper morphemes are interpreted as concrete grammatical and lexical informations. Below, there is a tentative list of proper morphemes belonging to the proper morphological dimensions G1 - G9 and L1 - L16 enumerated above :

G1 Singularis, Pluralis, Dualis, Trialis, Paucalis
G2 Masculinum, Femininum, Neutrum
G3 Praesent, Praeterit, Futuritas, Plusquamperfectum
G4 Prima, Secunda, Tertia
G5 Affirmativus, Negativus, Interrogativus, Imperativus, Hortativus, Iussivus, Optativus, Desideratívus, Prohibitivus, Conditionalis, Potentialis, Irrealis, Dubitativus, Debitivus, Permissivus, Habilitivus, Necessativus
G6 Nominativus, Accusativus, Dativus, Stativus, Instrumentalis, Destinativus, Comitativus, Essivus, Abessivus, Causalis, Finalis, Partitivus, Totivus, Genetivus, Comparativus, Locativus, Directivus, Vocativus
G7 Positivus, Maiorativus, Superativus, Excessivus
G8 Intransitivus, Transitivus, Reflexivus, Medialis, Reciprocalis
G9 Perfectivus, Imperfectivus, Ingressivus, Egressivus, Durativus, Iterativus, Frequentativus

L1 Denomination, Indication, Arrangement
L2 Apellativum, Nomen proprium
L3 Human, Animal, Plant, Object, Property, Quality, Action, State,...
L4 Concreteness, Abstractness
L5 Natural, Artificial
L6 Individuum, Genus
L7 Adult, Immature
L8 Father, Son, Mother, Brother, Sister, ...
L9 Male, Female
L10 Round, Rectangular, Oval, Elongated, ...
L11 Deminutativity, Augmentativity, Neutrality
L12 White, Black, Red, Blue, ...
L13 Mobility, Stativity
L14 **Large velocity, Small velocity**
L15 Hypocoristic, Augmentative
L16 Ameliorativity, Pejorativity.

The above listed proper morphemes are presented tentatively - particular languages choose some of those proper morphemes in any proper morphological dimension /GENUS in German consists of Masculinum, Femininum and Neutrum and GENUS in Italian consists of Masculinum and Femininum/. Moreover, in certain languages it is convenient to glue together some of

those proper morphemes /as in the case of MODUS, which may be considered as a set consisting of Indicativus, Interrogativus, Commendativus and Coniunctivus/. On the other hand, in certain languages it is necessary to crumble some of those proper morphemes in particular dimensions /cf. CASUS, where for instance Locativus may be replaced by such proper morphemes as Inessivus, Adessivus, Superficialis, Anteessivus, Posteessivus, Superessivus, Subessivus, Interessivus, Circumessivus and Temporalis/.

The next concept to be interpreted in linguistic terms is that of an actual lex. This is the fundamental unit of the plane of expression ; all other units of this plane /i.e. actual meaningful segments such as actual vocables and actual syntagmas/ are either parts of actual lexes or larger units, consisting of actual lexes. Also, several abstract linguistic entities /vocables, lexes, syntagmas, etc./ are constructed with the essential use of the concept of an actual lex.

Briefly, actual lexes are understood as minimal units complete with respect to lexical and grammatical information. Hence, the notion closest to actual lex considered in traditional linguistics is that of a word. However, there are slight differences between the two notions. Probably, the best way to explain the notion of an actual lex is to give examples of actual lexes in several languages. Thus, consider a sentence "He is coming to the library today" with its counterparts in Polish, Finnish, Hungarian, Chinese and Japanese :

He is coming to the library today.
Dzisiaj on przychodzi do biblioteki.
Hän tulee kirjastoon tänään.
Ő ma bejön a könyvtárba.
Jīntiān tā dào túshūguǎn lai.
Kare wa toshokan ni kyoo kimasu.

The above expressions are divided into actual lexes in the following way:

he, is coming, to the library, today
dzisiaj, on, przychodzi, do biblioteki
hän, tulee, kirjastoon, tänään
ő, ma, bejön, a könyvtárba
jīntiān, tā, dào lai, túshūguǎn
kare wa, toshokan ni, kyoo, kimasu

/this example is taken from BAŃCZEROWSKI 1980, p. 36 ; the borrowing of an example from the determinational theory of language is justified by the fact that our approach shares with that theory the same intuitions concerning the concept of an actual lex/.

Another thing which should be stressed in connection with the notion of an actual lex is that actual lexes are tokens, i.e. spatio-temporally fixed segments of language utterances. Thus every actual lex is uttered only once, by a given speaker in a fixed situation. We think that it is convenient to start a linguistic theory with tokens, as we share the opinion that any segment of the spatio-temporally fixed utterance conveys a fixed nonambiguous meaning. Abstract entities - types - can be then

constructed as suitable sets of concrete, i.e. actual, entities.

The extensional character of our approach to morphology is present in the fact that morphemes are defined to be sets of actual lexes. This means that we identify a morpheme expressing a given /lexical or grammatical/ information with the class of all actual lexes which convey this information. In particular, given any proper morphological dimension, the indefinite morpheme of this dimension consists of all those actual lexes which do not belong to proper morphemes of this proper morphological dimension, i.e. which do not contain information concerning this proper morphological dimension. For instance, the indefinite morpheme of the proper morphological dimension TEMPUS in English consists of all English actual lexes except verbs.

We think that the interpretation of morphological dimensions follows from the definition and the formal properties of this concept described in the above propositions.

At the end of this section let us introduce a few concepts which will be useful in the following sections.

2.16. Definition. For any actual lex $x \in Lex$ let us define :

$mp(x) = \{\Sigma \in mph : x \in \Sigma\}$ $\quad dm(x) = \{\mathfrak{A} \in dim : \mathfrak{A} \cap mp(x) \neq \emptyset\}$

$Mp(x) = \{\Sigma \in Mph : x \in \Sigma\}$ $\quad Dm(x) = \{\mathfrak{A} \in Dim : \mathfrak{A} \cap Mp(x) \neq \emptyset\}$.

A few simple properties of these concepts are presented in the next propositions.

2.17. Proposition. For every actual lex $x \in Lex$, the following inclusions hold : $mp(x) \subset Mp(x)$, $\bigcap Mp(x) \subset \bigcap mp(x)$.

2.18. Proposition. For any actual lexes $x,y \in Lex$, if $mp(x) = mp(y)$, then $Mp(x) = Mp(y)$.

2.19. Proposition. For any actual lexes $x,y \in Lex$, if $dm(x) = dm(y)$, then $Dm(x) = Dm(y)$.

2.20. Proposition. For any actual lexes $x,y \in Lex$, if $mp(x) = mp(y)$, then $dm(x) = dm(y)$.

3. Hyponymy

This section introduces a new primitive concept, viz. that of hyponymy hpn. This concept, as well as some concepts defined by it, is characterized by three axioms.

Traditionally, hyponymy is understood as a relation between lexical units : x is said to be a hyponym of y, if the meaning /denotation/ of x is narrower than the meaning /denotation/ of y. Thus, for instance, 'rose' is a hyponym of 'flower', 'steal' is a hyponym of 'get' and 'scarlet' is a hyponym of 'red'. Lexical hyponymy can be defined for instance by unilateral implication between sentences /cf. LYONS 1977, p. 292/.

It seems difficult /if possible at all/ to extend the relation of hyponymy, understood as above, to expressions larger than lexical units. For this reason, it is more plausible to consider the relation of ex-

tended hyponymy as based on content of expressions rather than on their denotation. Thus, we propose to understand hyponymy as a relation holding between expressions x and y if and only if the content of x is more comprehensive than the content of y /the notion of content is understood here in an intuitive way - cf. the examples of actual meaningful segments within the relation of hyponymy given below/.

AXIOM 2. $hpn \in MPO(eln) \cap FU \wedge eqv(hpn) \in FI(fld(hpn), P\langle Lex\rangle)$

3.1. Definition. $Sgm = fld(hpn)$. Elements of the set Sgm are called actual meaningful segments.

For any actual meaningful segments x and y, if x hpn y, then we say that x is a hyponym of y, or, equivalently, that y is a hyperonym of x. Below, there are examples of actual meaningful segments which are within the relation of hyponymy :

'steal' hpn 'get'
'scarlet' hpn 'red'
'rose' hpn 'flower'
'black dog' hpn 'dog'
'is moving quickly' hpn 'is moving'
'the boy is going to the school' hpn 'the boy'
'he is coming to the library today' hpn 'he is coming to the library'

By definition, actual meaningful segments are parts of linear objects. In this way we reflect the linear character of speech in our axiomatics.

A few simple but fundamental properties of hyponymy are presented in the next proposition.

3.2. Proposition.

a/ Any actual meaningful segment is a hyponym of itself.

b/ For any actual meaningful segments x,y,z, if x is a hyponym of y and y is a hyponym of z, then also x is a hyponym of z.

c/ For any actual meaningful segments x and y, if x is a part of y, then x is not a hyponym of y.

d/ The set Sgm does not contain any infinite chain $x_0, x_1, x_2, \ldots$ such that x_i hpn x_{i+1} for $i \geqslant 0$ and $x_i \neq x_j$ for $i \neq j$.

e/ For any actual meaningful segment x there exists an actual meaningful segment y such that x hpn y and there is no actual meaningful segment $z \neq y$ for which y hpn − conv(hpn) z.

Before formulating axioms 3 and 4 let us introduce a few new concepts.

3.3. Definition. $Stg = \{x \in Sgm : x = S(Lex \cap P^{\vee}x) \wedge card(Sgm \cap P^{\vee}x)$ is finite $\wedge\ card(Lex \cap P^{\vee}x) \geqslant 2 \wedge$ there exists exactly one $y \in Lex \cap P^{\vee}x$ such that x hpn y$\}$. Elements of the set Stg are called actual syntagmas.

3.4. Definition. $Vcb = \{x \in (Sgm \cap ln) - mo : \neg \bigvee_z (z \in Sgm \wedge z\ P\ x \wedge z \neq x\}$.

Elements of the set Vcb are called actual vocables.

3.5. Definition. $Ut = \{x \in Stg : \neg \bigvee_y (y \in Stg \wedge y \neq x \wedge x\ P\ y\}$. Elements

of the set Ut are called actual utterances.

3.6. Definition. $hsg = eqv(hpn)$. If x hsg y, then we say that x and y are homosignificative.

3.7. Definition. For any actual meaningful segment x, let us define :

$sgm(x) = Sgm \cap P^{\vee}x$ $\quad$ $vcb(x) = Vcb \cap P^{\vee}x$

$lex(x) = Lex \cap P^{\vee}x$ $\quad$ $stg(x) = Stg \cap P^{\vee}x$.

AXIOM 3. $Sgm = Vcb \cup Lex \cup Stg$

AXIOM 4. $(Ut, Lex, Vcb) \in HFP$

A few words about the linguistic interpretation of the sets Vcb, Stg and Ut are in order. Actual vocables are interpreted as minimal meaningful free forms /i.e. words in the sense of Bloomfield/. Given an arbitrary actual meaningful segment x, the set of all actual vocables being parts of x equals vcb(x). Thus, we have :

vcb('he is coming to the library today') = {'he','is','coming','to', 'the', 'library','today'}

vcb('hän tulee kirjastoon tänään') = {'hän','tulee','kirjastoon','tänään'}.

As the reader surely observed we admit the possibility that a given actual meaningful segment is simultaneously an actual lex and an actual vocable. In this way, the traditional concept of a word is split into two concepts in our approach.

Actual syntagmas are interpreted as syntactically complex entities. Essential constituent parts of actual syntagmas are actual lexes. The fundamental property of actual syntagmas is their semantically endocentric character : any actual syntagma is a hyponym of exactly one of its constituent actual lexes. Thus, actual syntagmas satisfy, in a certain sense, Frege Principle which says that the meaning of a complex expression is a function of the meanings of its parts. Here are a few examples of actual syntagmas in English /in each case we specify which actual lex is the hyperonym of a given actual syntagma/ :

'the oldest son'
'he is coming to the library'
'Mary is walking in the garden'
'is running very quickly'.

It is worth pointing out that all actual meaningful segments /i.e. actual vocables, actual lexes and actual syntagmas/ are tokens. As we will see from the propositions of this section, all actual meaningful segments are parts of actual utterances. Actual utterances are maximal tokens - this follows from definition 3.5. and axiom 4.

A remark concerning axiom 3 is necessary. Namely, it follows from this axiom that actual vocables, actual lexes and actual syntagmas exhaust the set of all meaningful segments. It should be kept in mind, however, that the set Sgm has to be interpreted as a certain subspace of the space of all language expressions /i.e. we do not consider all possible language

expressions/.

Now, let us present some simple consequences of axioms 2, 3 and 4.

3.8. Proposition. $Sgm \cap P\langle Lex\rangle = Lex \cup Vcb$

3.9. Proposition. The sets $(Lex \cup Vcb)/hsg$, Lex/hsg and Vcb/hsg are all finite.

3.10. Proposition. No two distinct actual utterances have any common parts.

3.11. Proposition. No two distinct actual lexes have any common parts.

3.12. Proposition. No two distinct actual vocables have any common parts.

3.13. Proposition. Any actual vocable is a part of exactly one actual lex.

3.14. Proposition. Any actual meaningful segment is a part of exactly one actual utterance.

3.15. Proposition. For any actual syntagma x, the set $lex(x)$ is a mereological partition of x.

3.16. Proposition. For any actual meaningful segment $x \in Lex \cup Stg$, the set $vcb(x)$ is a mereological partition of x.

3.17. Proposition. Every actual lex is a part of some actual syntagma.

3.18. Proposition. $S(Ut) = S(Stg) = S(Lex) = S(Vcb) = S(Sgm)$.

3.19. Proposition. For any actual utterance u, $u = S(stg(u)) = S(lex(u)) = S(vcb(u)) = S(sgm(u))$.

3.20. Proposition. For any two distinct actual syntagmas x and y, either $P^{\vee}x \cap P^{\vee}y = \emptyset$ or there is exactly one actual utterance z such that both x and y are parts of z.

3.21. Proposition. For any two actual syntagmas x and y, if x is a part of y, then $lex(x) \subset lex(y)$ and $vcb(x) \subset vcb(y)$.

3.22. Proposition. For any two actual syntagmas x and y, if $P^{\vee}x \cap P^{\vee}y \neq \emptyset$, then $x \frown y = S(lex(x) \cap lex(y))$.

3.23. Proposition. The sets Stg, Ut and Vcb are all non-empty.

3.24. Proposition. $Stg \cap Lex = Stg \cap Vcb = \emptyset$

3.25. Proposition. $Sgm \cap mo = \emptyset$

3.26. Proposition. If x is an actual syntagma, then $stg(x) \neq \emptyset$, $lex(x) \neq \emptyset$ and $vcb(x) \neq \emptyset$.

3.27. Proposition. If x is an actual lex, then $stg(x) = \emptyset$, $lex(x) - \{x\} = \emptyset$ and $vcb(x) \neq \emptyset$.

3.28. Proposition. If x is an actual vocable, then $sgm(x) - \{x\} = \emptyset$.

3.29. Proposition. For any actual meaningful segment x, the set $vcb(x)$ is linearly ordered by the relation T_e /i.e. T_e is a connected partial ordering of $vcb(x)$/.

3.30. Proposition. For any actual meaningful segment x, the set $sgm(x)$ is finite.

We think that the above propositions gave the reader a more less clear idea how the set of all actual meaningful segments looks like. Some further properties of hyponymy will be presented in the next sections.

4. Homophony

The relation of homophony hph will be used /together with the relation of homosignification hsg/ for the construction of types out of tokens. Intuitively, homophony holds between those actual meaningful segments which are indistinguishable with respect to their phonetic form.

AXIOM 5. $hph \in MC(Sgm) \cap FI(Sgm, Lex \cup Vcb)$

If x hph y, then we say that x and y are homophonous. Here are some examples of homophonous actual meaningful segments in English :

x hph x for any actual meaningful segment x

'the sons raise meat' hph 'the suns rays meet'

'lean' /verb/ hph 'lean' /adjective/

'left' /past participle/ hph 'left' /adjective/.

4.1. Definition. $eq = hsg \cap hph$. By definition, eq is an equivalence relation. Hence we can define $LEX = Lex/eq$ and $VCB = Vcb/eq$. Elements of the set LEX are called lexes and elements of the set VCB are called vocables.

If $x \in Sgm$, then $eq(x)$ is the set of all actual meaningful segments which are homophonous and homosignificative with x. Thus, elements of the sets LEX and VCB are types, i.e. abstract entities belonging to langue and not to parole. At the moment, we are not going to consider types of syntactically complex expressions /cf. section 6/.

4.2. Proposition. The sets $(Lex \cup Vcb)/hph$, Lex/hph, Vcb/hph, LEX and VCB are all finite.

4.3. Proposition. For any actual meaningful segments x,y,u,v, if $x \cup y$, $x \dot{-} y$, $u \cup v$, $u \dot{-} v \in Sgm$, x hph u and y hph v, then also $x \cup y$ hph $u \cup v$ and $x \dot{-} y$ hph $u \dot{-} v$.

4.4. Proposition. Actual meaningful segments x and y are homophonous if and only if there exists a function $f : vcb(x) \longrightarrow vcb(y)$ such that :

a/ f is one-one and onto

b/ u hph $f(u)$ for every $u \in vcb(x)$

c/ $u\ T_i\ v$ if and only if $f(u)\ T_i\ f(v)$ for every $u,v \in vcb(x)$.

The above proposition characterizes the relation of homophony rather strongly. Among others, it has the following consequences :

4.5. Proposition. If $x \in Vcb$, then $hph(x) \subset Vcb$ and $eq(x) \subset Vcb$.

4.6. Proposition. If $x \in Lex$, then $hph(x) \subset Lex$ and $eq(x) \subset Lex$.

4.7. Proposition. If $x \in Stg$, then $hph(x) \subset Stg$.

The next axiom, as axiom 5, connects all three primitive concepts of our system.

AXIOM 6. $\bigwedge_x \bigwedge_y (x \in Lex \wedge y \in Lex \longrightarrow (x\ eq\ y \longleftrightarrow mp(x) = mp(y)))$

This axiom says that two actual lexes are within the relation eq /i.e.