WHY DO WE BELIEVE IN THE INTENDED MODELS?

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The origin of many mathematical theories is related to the reflection on properties of some distinguished structures. To be slightly more precise: the presently accepted important axiomatic theories are final results of a long research during which we have accumulated the knowledge about some structures thought of later as standard. Up to the XIXth century the axiomatic method in an overt form was applied in geometry only. Due to efforts of several mathematicians from the XIXth and early XXth century it has become standard. The awareness that a mathematical theory can have many different models arose subsequently. Simultaneous development of mathematical logic has brought new concepts – like *categoricity* and *completeness* – as well as (mainly negative) results concerning them, e.g. the Löwenheim-Skolem theorem, incompleteness results in arithmetic and set theory. On the basis of these results it can be argued that the notion of the *intended model* of a theory can be characterized mainly in the *pragmatic* sense. However, professional mathematicians seem to be satisfied with characterization of their beloved *intended* models (like the standard natural numbers, the real numbers, the "true" sets), though such characterization is possible only in rather strong systems of logic, lacking some nice deductive properties. Thus, it seems that they strongly believe in the intended models, so to speak. There are some theorems (like e.g. Tennenbaum theorem, isomorphism theorems in algebra, etc.) which support such beliefs. Nevertheless the initial hope that some simple *extremal axioms* could provide the unique characterization of the intended models underwent revision which is visible for instance in the modern model theory.

We are going to discuss some aspects of the approach to the intended models in a historical perspective in our talk, stressing the role of extremal axioms in this development.

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