Cognitive Accessibility of Mathematical Objects

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Jaroslav Hašek wrote that it is difficult to describe non-existing animals but it is much more harder to show them to the audience. We are not going to discuss the old dilemma: is mathematics *created* or *discovered*? Rather, we will focus our attention on the *access* which we have to mathematical objects themselves. Moreover, this access will be characterized *inside* mathematics and not based on, say, philosophical considerations about perception.

In turn, John von Neumann expressed the opinion that in mathematics we are not aiming at *understanding* it but we rather *get accustomed* to it. Is this *dictum* a play with words only? We think of mathematics as a *science of patterns* which is also an *art of solving problems*, according to prescribed rules. The *meanings* of mathematical concepts are determined by the underlying theory. *Understanding* these concepts is obtained in the *context of transmission* of mathematical knowledge, with the help of *intuitive explanations*.

Mathematical objects can be *standard*, *exceptional* or *pathological*. Whether they are considered as *well-behaving* depends on the goals they are supposed to serve. *Domestication* of mathematical objects is a result of accumulation of knowledge about them and widening the scope of their applications.

The objects of each mathematical domain may be classified with respect to their *accessibility* for the cognitive subject. We have different cognitive access to several sorts of numbers: integers, rational, algebraic, constructible, computable, irrational, transcendental, normal, etc. numbers. There are *easy* sets (finite, Borel, constructible) and *difficult* ones (Bernstein, Cantor, Vitali, large cardinals, indecomposable continua). Functions are classified in the Baire's hierarchy. One can distinguish degrees of *computability* and *incomputability*. Some examples will be discussed in details in our talk.

We are going to emphasize the fact that *degrees of accessibility* of mathematical objects can be characterized in mathematics itself, thus without support of metaphysical assumptions. However, the accessibility in question is relativized historically and depends on the expressive power of mathematical discourse, as we will try to show.

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