Semiotyka logiczna

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Dodatek 15.2.

Triada hipotaktyczna: część 1

Niniejsza prezentacja zawiera tekst artykułu ***An axiom system for hypotaxis***, opublikowanego w 1981 roku w serii preprintów *Working Papers of the Institute of Linguistics, Adam Mickiewicz University, Poznań*.

W artykule podaje się m.in. system aksjomatów dla determinacyjnej teorii języka, sformułowanej przez Profesora Jerzego Bańczerowskiego.

Triada hipotaktyczna: część 1

System ten był później badany m.in. w następujących pracach piszącego te słowa:

- *Grundideen der kombinatorischen Semantik.* SAIS Arbeitsberichte, Christian-Albrechts-Universität, Kiel 1985, 133-152. [Zobacz Dodatek 16.]
- *Hiponimia.* Wydawnictwo Naukowe UAM, Poznań 1991. [Zobacz Dodatek 17.]
- *Combinatory semantics.* Wydawnictwo Naukowe UAM, Poznań 1993.

Ze względu na dużą objętość pliku, został on podzielony na dwa fragmenty. To jest drugi fragment.

belong to the same lex/ if and only if they belong to exactly the same proper morphemes. Consequently, if x and y belong to the same lex, then they belong to exactly the same morphemes and dm(x) = dm(y) as well as Dm(x) = Dm(y). Axiom 6 also implies that the following definition is correct :

4.8. <u>Definition</u>. The function MB : $LEX \longrightarrow \mathcal{P}(Mph)$ is defined for any $x \in Lex$ by MB(eq(x)) = Mp(x). The set MB(X) is called the <u>morphemic bundle</u> of the lex X.

4.9. <u>Proposition</u>. No two different lexes have the same morphemic bundle.

This proposition expresses the fact that the set of all proper morphological dimensions is a maximally economical description of the semantic structure of the lexical universe.

4.10. <u>Proposition</u>. $eq(x) \subset \bigcap mp(x)$ for every $x \in Lex$.

5. <u>Hypotaxis</u>

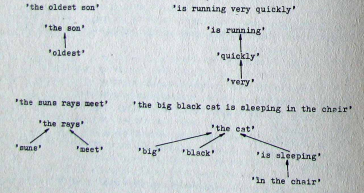
This section introduces the main syntactic concept of the work, i.e. that of hypotaxis. As hypotaxis is defined by hyponymy, we get a semantically based description of a part of syntax in this way.

5.1. <u>Definition</u>. For any actual syntagma x, let hpr(x) denote the only actual lex y such that y P x and x hpn y.

Definition 5.1. is correct by 3.3.

5.2. <u>Definition</u>. hpt = $\{(x,y) : x,y \in Lex \wedge x \dot{\smile} y \in Stg \wedge y = hpr(x \dot{\smile} y)\}$. The relation hpt is called <u>hypotaxis</u>. If x hpt y, then x is called <u>hypotactically subordinate</u> and y is called <u>hypotactically superordinate</u>.

Consider a few examples of English actual syntagmas in which actual lexes are connected by the relation of hypotaxis. Here an arrow between actual lexes x and y means that x hpt y.



<u>AXIOM</u> <u>7</u>. $(\{(lex(x), hpr(x)) : x \in Stg\}, hpt, eq) \in UFO$

This is the last axiom of our system. Let us present a few simple consequences of it.

5.3. <u>Proposition</u>. $hpt \cap eq = \emptyset$. Consequently, if $x,y \in Lex$ and $x \dot{\smile} y \in Stg$, then x eq y does not hold.

5.4. <u>Proposition</u>. For any actual lexes x,y,u,v, if x eq u, y eq v and x hpt y, then u hpt v.

5.5. <u>Proposition</u>. The system $(lex(x), hpt \cap lex(x)^2, hpr(x))$ is a reduced tree for any actual syntagma x.

5.6. <u>Proposition</u>. For any actual lexes x and y, if x hpt y, then :

a/ $x \neq y$

b/ y hpt x does not hold

c/ there is no $z \in Lex$ such that $z \neq x$, $z \neq y$, x hpt z and z hpt y.

5.7. <u>Proposition</u>. For any actual lexes x, y and z :

a/ if x hpt y and y hpt z, then neither x hpt z nor z hpt x

b/ if x hpt y and x hpt z, then y = z.

5.8. <u>Proposition</u>. $\{hpt \cap lex(x)^2 : x \in Ut\}$ is a partition of hpt.

5.9. <u>Proposition</u>. For any actual lex x there exists an actual lex y such that x hpt y or y hpt x.

5.10. <u>Proposition</u>. For any actual lex x there exists an actual lex y such that $x \dot{\smile} y$ is an actual syntagma.

5.11. <u>Proposition</u>. $hpt \neq \emptyset$ and $hpn \neq \emptyset$.

5.12. <u>Proposition</u>. For any actual lexes x,y,z, if $S(\{x,y,z\}) \in Stg$, then $\{x \dot{\smile} y, x \dot{\smile} z, y \dot{\smile} z\}$ is nót contained in Stg.

5.13. <u>Proposition</u>. If $x \in Stg$, $u \in Ut$ and x P u , then the system $(lex(x), hpt \cap lex(x)^2, hpr(x))$ is a subtree of $(lex(u), hpt \cap lex(u)^2, hpr(u))$.

6. <u>Hypotactical</u> <u>structures</u>

We discuss in this section several structures defined with the help of the concepts introduced in previous sections. Our presentation of these topics is, however, rather brief. We limit ourselves to the indication of directions in which our axiom system can be developed. We discuss subsequently : non-linear /hypotactical/ structures, linear structures, some structures occurring in the lexical universe, and, finally, the problem of connections between hypotaxis and the theory of formal grammars.

The relation of hypotaxis holds between certain actual meaningful segments, viz. actual lexes occurring in actual syntagmas. It is possible to introduce the analogon of this relation for abstract entities - lexes.

6.1. <u>Definition</u>. We say that actual syntagmas x and y are <u>hypotactically equivalent</u>, in symbols x heq y, if there exists a function f : $lex(x) \longrightarrow lex(y)$ such that :

a/ f is one-one and onto

b/ u hpt v if and only if f(u) hpt f(v) for every $u, v \in \mathrm{lex}(x)$

c/ z eq f(z) for every $z \in \mathrm{lex}(x)$.

Of course, heq is an equivalence relation. Observe that if x heq y, then the corresponding hypotactical reduced trees are isomorphic. Moreover, it follows from 3.3. and 4.2. that each isomorphism-equivalence class contains only a finite number of heq-equivalence classes.

6.2. <u>Definition</u>. STG = Stg/heq. Elements of the set STG are called <u>syntagmas</u>.

We see that each syntagma is a maximal class of hypotactically equivalent actual syntagmas. In other words, two actual syntagmas belong to the same syntagma if and only if they are indistinguishable with respect to hypotactical structure as well as to lexical content.

6.3. <u>Definition</u>. Let the relation $\mathrm{Hpt} \subset \mathrm{LEX}^2$ be defined as follows : eq(x) Hpt eq(y) if and only if x hpt y for any $x, y \in \mathrm{Lex}$.

The correctness of this definition follows from 5.4. The relation Hpt is a counterpart of hypotaxis for lexes.

Any syntagma can be associated a reduced tree in the following way. If A is a syntagma, then consider a graph isomorphic to any of the reduced trees of actual syntagmas from A whose vertices are eq-equivalence classes of actual lexes occurring in those actual syntagmas /the uniqueness of this construction follows from 6.1. and 6.2./.

Using hypotactical structures determined on the set of lexes /i.e. trees associated with syntagmas/ one can investigate local syntactic properties of complex expressions /for example, hypotactical neighbourhoods of lexes in hypotactical trees/ as well as some global structures in the lexicon /for example, hypotactical categories of lexes determined by their occurrence in hypotactical syntagmatic trees/.

Linear structures of actual meaningful segments are determined by the mereological relation of precedence in time /cf. 3.29./. Now, we will show how to introduce linear structures into abstract complex expressions.

6.4. <u>Definition</u>. Let VCB^+ denote the free semigroup generated by VCB, i.e. VCB^+ is the set of all finite strings of vocables /including the empty string ε/. The symbol VCB_ω stands for the set of all finite sequences /ordered tuples/ of vocables. Let us define two functions $\mathrm{lin} : \mathrm{Sgm} \longrightarrow \mathrm{VCB}^+$ and $\mathrm{ln} : \mathrm{Sgm} \longrightarrow \mathrm{VCB}_\omega$ by :

$$\mathrm{lin}(x) = \mathrm{eq}(x_1)\,\mathrm{eq}(x_2)\ldots\mathrm{eq}(x_n) \qquad \mathrm{ln}(x) = (\mathrm{eq}(x_1), \mathrm{eq}(x_2), \ldots, \mathrm{eq}(x_n))$$

for $x \in \mathrm{Sgm}$, where $\mathrm{vcb}(x) = \{x_1, x_2, \ldots, x_n\}$ and $x_j \; T_i \; x_{j+1}$ for $1 \leqslant j < n$. Let Lin = lin[Stg] and LIN = lin[Ut]. For any vocable $X \in \mathrm{VCB}$, if $aXb = c \in \mathrm{LIN}$ for some $a, b \in \mathrm{VCB}^+$, then let $\mathrm{cnt}_c(X) = (a, b)$ /<u>context</u> of X in c/. For any $a, b \in \mathrm{VCB}^+$, we say that a is a <u>subword</u> of b, in symbols a sbw b, if and only if b = cad for some $c, d \in \mathrm{VCB}^+$.

The concepts introduced in 6.4. are well known from mathematical linguistics. We have to present some generalizations of them, because it is necessary to have analogons of contexts for lexes which do not coincide

with vocables /lexes may be considered as sequences of vocables/.

6.5. <u>Definition</u>. If $a \in \mathrm{VCB}^+$ and $\beta = (b_1, \ldots, b_k)$, $b_i \in \mathrm{VCB}^+$, then we say that β <u>alternates</u> a, in symbols β alt a, if and only if :

$$a = a_1 b_1 a_2 b_2 \ldots a_k b_k a_{k+1} \quad \text{and} \quad a_i \in \mathrm{VCB}^+ \quad \text{for} \quad 1 \leqslant i \leqslant k+1.$$

The next proposition shows that alternation is a generalization of the relation of being a subword.

6.6. <u>Proposition</u>. For any $a, b \in \mathrm{VCB}^+$, the one-element sequence (b) alternates a if and only if b is a subword of a.

It follows from this proposition that (a) alt a for any $a \in \mathrm{VCB}^+$. Some further simple properties of alternation are described in the next propositions.

6.7. <u>Proposition</u>. For any actual meaningful segments x and y, lin(x) = lin(y) if and only if ln(x) = ln(y).

6.8. <u>Proposition</u>. For any actual meaningful segments x and y, if x P y, then ln(x) alt lin(x).

6.9. <u>Proposition</u>. For any actual meaningful segment x there exists $a \in \mathrm{LIN}$ such that ln(x) alt a .

The relation of alternation can be used for the definition of the concept of context applicable for lexes.

6.10. <u>Proposition</u>. Let $a \in \mathrm{VCB}^+$. For $\beta = (b_1, \ldots, b_k)$ such that β alt a, define $a \mathbin{\hat{-}} \beta$ to be the only γ for which $\gamma = (c_1, \ldots, c_{k+1})$, $c_i \in \mathrm{VCB}^+$ and $a = c_1 b_1 c_2 b_2 \ldots c_k b_k c_{k+1}$.

The correctness of 6.10. follows from 6.5. and some general properties of free semigroups.

6.11. <u>Definition</u>. For $X \in \mathrm{LEX}$ and $a \in \mathrm{LIN}$ such that ln(X) alt a, the sequence $a \mathbin{\hat{-}} \mathrm{ln}(X)$ is called the <u>context</u> of X in a. Two lexes X and Y are <u>homodistributive</u>, in symbols X hdb Y, if $a \mathbin{\hat{-}} \mathrm{ln}(X) = a \mathbin{\hat{-}} \mathrm{ln}(Y)$ for all $a \in \mathrm{LIN}$ such that either ln(X) alt a or ln(Y) alt a. The context of X in a, where $X \in \mathrm{LEX}$ and $a \in \mathrm{LIN}$, is denoted by $\mathrm{con}_a(X)$.

Consider two examples of contexts of lexes :

$\mathrm{con}_{\text{'the black c.. is sleeping'}}$('black') = ('the','cat is sleeping')

$\mathrm{con}_{\text{'all windows of the biggest building are open'}}$('of the building') = ('windows', ε ,'biggest','are open').

Observe that for any $x \in \mathrm{Lex} \cap \mathrm{Vcb}$ and any $a \in \mathrm{LIN}$ we have $\mathrm{cnt}_a(\mathrm{eq}(x)) = \mathrm{con}_a(\mathrm{eq}(x))$. This shows that 6.11. is an adequate generalization of the classical concept of context.

Lexes can be grouped together into several sets with respect to their occurrences in linear structures of complex expressions. One of such partitions is that into homodistribution-equivalence classes. Some other groupings are presented in the next definition.

6.12. <u>Definition</u>.

a/ $\mathrm{dLEX} = \{X \in \mathrm{LEX} : \bigvee_a (a \in \mathrm{LIN} \wedge \mathrm{ln}(X) \text{ alt } a \wedge \neg \mathrm{lin}(X) \text{ sbw } a)\}$ /<u>discontinual lexes</u>/

b/ cLEX = LEX − dLEX /<u>continual lexes</u>/

c/ $\mathrm{sdLEX} = \{X \in \mathrm{LEX} : \bigwedge_a (a \in \mathrm{LIN} \longrightarrow (\mathrm{ln}(X)\ \mathrm{alt}\ a \longrightarrow \neg \mathrm{lin}(X)\ \mathrm{sbw}\ a))\}$

/<u>strictly</u> <u>discontinual</u> <u>lexes</u>/.

We have for instance 'the cat' $\in$ dLEX, because 'the black cat is sleeping' $\in$ LIN. The lex 'for sure' is an axample of continual lex in English. It is not clear, whether the set of strictly discontinual lexes in English is non-empty.

The next definition connects linear structures with hypotactical /non-linear/ structures.

6.13. <u>Definition</u>. A string $a \in \mathrm{Lin}$ is <u>hypotactically</u> <u>ambiguous</u>, if there are $x, y \in \mathrm{Stg}$ such that $\mathrm{lin}(x) = \mathrm{lin}(y) = a$ and x heq y does not hold /i.e. x and y do not belong to the same syntagma/.

A string 'they are flying planes' is hypotactically ambiguous : one can find two actual syntagmas x and y such that $\mathrm{lin}(x) = \mathrm{lin}(y) =$ = 'they are flying planes' and x and y are not hypotactically equivalent. The reduced trees associated with the actual syntagmas in question are presented below :

$$\begin{tikzcd}[row sep=small] \text{'they'} \\ \text{'are'} \arrow[u] \\ \text{'planes'} \arrow[u] \\ \text{'flying'} \arrow[u] \end{tikzcd} \qquad\qquad \begin{tikzcd}[row sep=small] \text{'they'} \\ \text{'are flying'} \arrow[u] \\ \text{'planes'} \arrow[u] \end{tikzcd}$$

One can investigate several structures determined by morphological constructions introduced in section 2.

6.14. <u>Definition</u>. Let us define the relation $\mathrm{mc} \subset \mathrm{LEX} \times \mathrm{Mph}$ by $X\ \mathrm{mc}\ \Sigma$ if and only if $\Sigma \in \mathrm{MB}(X)$. The set MC of all <u>morphological</u> <u>categories</u> is defined as follows : $\mathrm{MC} = \{\mathrm{mc}\langle\mathfrak{A}\rangle : \mathfrak{A} \subset \mathrm{Mph}\}$.

Observe that for any $\mathfrak{A} \subset \mathrm{Mph}$: $\mathrm{mc}\langle\mathfrak{A}\rangle = \bigcup\{\mathrm{mc}\langle\{\Sigma\}\rangle : \Sigma \in \mathfrak{A}\}$.

6.15. <u>Proposition</u>. $\{\mathrm{eq}(x)\} = \bigcap\{\mathrm{mc}^{\vee}\Sigma : \Sigma \in \mathrm{Mp}(x)\}$ for every actual lex $x \in \mathrm{Lex}$.

6.16. <u>Proposition</u>. $\mathrm{mc}\langle\mathrm{Mph}\rangle = \bigcup\{\mathrm{mc}^{\vee}\Sigma : \Sigma \in \mathrm{Mph}\} = \bigcup\{\mathrm{mc}^{\vee}\Sigma : \Sigma \in \mathrm{mph}\} =$
$= \mathrm{mc}\langle\mathrm{mph}\rangle = \mathrm{LEX}$.

6.17. <u>Proposition</u>. For every morphological dimension $\mathfrak{A} \in \mathrm{Dim}$, the set $\{\mathrm{mc}^{\vee}\Sigma : \Sigma \in \mathfrak{A}\}$ is a partition of LEX.

We recall that by a <u>similarity</u> /<u>tolerance</u>/ relation we understand any relation which is reflexive and symmetric. An ordered pair consisting of a non-empty set and a tolerance on it is called a <u>tolerance</u> <u>space</u>. Tolerance relations are formal counterparts of similarity and partial indistinguishability.

6.18. <u>Definition</u>. For any set of morphemes $\mathfrak{A} \subset \mathrm{Mph}$, let us define the relation $\mathrm{sm}_{\mathfrak{A}} \subset \mathrm{LEX} \times \mathrm{LEX}$ by $X\ \mathrm{sm}_{\mathfrak{A}}\ Y$ if and only if $\mathfrak{A} \subset \mathrm{MB}(X) \cap \mathrm{MB}(Y)$ and let $\mathrm{sm} = \bigcup\{\mathrm{sm}_{\{\Sigma\}} : \Sigma \in \mathrm{Mph}\}$.

6.19. <u>Proposition</u>. (LEX,sm) is a tolerance space.

Several similarity relations based on hyponymy can also be defined on the set of all **actual meaningful** segments.

6.20. <u>Definition</u>. For any $x, y \in \mathrm{Sgm}$, let

$x\ \mathrm{sim}\ y \longleftrightarrow \mathrm{hpn}^{\wedge}x \cap \mathrm{hpn}^{\wedge}y \neq \emptyset$

$x\ \mathrm{sml}\ y \longleftrightarrow \mathrm{hpn}^{\vee}x \cap \mathrm{hpn}^{\vee}y \neq \emptyset$

$x\ \mathrm{inc}\ y \longleftrightarrow \neg(x\ (\mathrm{hpn} - \mathrm{conv}(\mathrm{hpn}))\ y) \wedge \neg(y\ (\mathrm{hpn} - \mathrm{conv}(\mathrm{hpn}))\ x)$.

6.21. <u>Proposition</u>. (Sgm,inc), (hpn⟨Sgm⟩,sim) and (hpn[Sgm],sml) are tolerance spaces.

The concepts introduced in 6.18. and 6.20. can be characterized with the help of the theory of tolerance spaces /cf. POGONOWSKI 1981, chapters 1,2,3,5/. Also, one can introduce analogons of the relations from 6.20. for lexes, using the following definition :

6.22. <u>Definition</u>. Let the relation $\mathrm{Hpn} \subset \mathrm{LEX} \times \mathrm{LEX}$ be defined by $\mathrm{eq}(x)\ \mathrm{Hpn}\ \mathrm{eq}(y)$ if and only if x hpn y for any $x, y \in \mathrm{Lex}$.

The correctness of this definition follows from 4.1.

6.23. <u>Proposition</u>. The relation Hpn is a partial ordering of the set of all lexes.

Some properties of the set Sgm can be investigated with the help of model-theoretic notions. For the purpose of the next definition, we assume that the reader is familiar with elementary model theory /cf. any good textbook on mathematical logic/. Consider the structure $\mathfrak{M} =$ = (Sgm,dim,hpn,hph) and let L be the language of set theory with P,T, dim,hpn,hph as the non-logical constant symbols. Denote by Fml the set of all formulas of L with one free variable.

6.24. <u>Definition</u>. Let $F \subset \mathrm{Fml}$. We say that two actual meaningful segments x and y are <u>F-synonymous</u> if and only if x and y satisfy in $\mathfrak{M}$ exactly the same formulas from F. For any $x \in \mathrm{Sgm}$, let <u>sense</u> of x be defined to be the set of all formulas from Fml which are satisfied by x in $\mathfrak{M}$.

6.25. <u>Proposition</u>. For any $F \subset \mathrm{Fml}$, the relation of F-synonymy is an equivalence.

6.26. <u>Proposition</u>. For any $x, y \in \mathrm{Sgm}$, sense of x is identical with sense of y if and only if x and y are Fml-synonymous.

As proposition 6.26. shows, Fml-synonymy coincides with the identity of sense /where sense of an actual meaningful segment x is understood as the place of x in the space of all actual meaningful segments/. This is in accordance with the widespread opinion concerning synonymy /cf. LYONS 1971, p. 427/. Observe that Fml-synonymy corresponds to the relation of synonymy in a strict, theoretical sense.

The last problem we are going to discuss in this section is that of connections between hypotaxis and the theory of formal grammars. This problem can be formulated as follows :

Find a formal grammar, generating the set LIN, whose syntactic structures are based on hypotaxis.

The formal grammar in question should be of the form $G = (SC, VCB, U, RR)$ where VCB is the terminal vocabulary, SC is the non-terminal vocabulary /the set of all syntactic categories/ containing the initial symbol U /utterance/ and RR is the set of rewriting rules. Syntactic categories should be defined with the help of hypotactical structures /such as hypotactical neighbourhoods, homodistribution, morphological categories/. The rewriting rules should provide for the connections between syntactic categories, lexes, syntagmas, vocables and the set LIN. It is most likely that a generalized version of dependency grammar /cf. HAYS 1964/ will be adequate for those purposes.

7. Extensions of the initial system

The last section of this work indicates a few possible generalizations of the present axiom system. We discuss briefly some problems concerning homolexy, signification comprehension and moods of expression.

In section 2, when discussing proper morphological dimensions, we did not make any formal distinction between lexical and grammatical information. In other words, we did not impose any internal structure into the set dim /besides that introduced in axiom 1/. Now, we are going to distinguish a certain subset of dim corresponding to dimensions of lexical information. The subset in question is intended to include, among others, proper morphological dimensions L1 - L16 listed in section 2.

Any language makes a distinction between lexical and grammatical information - this is true even of "purely isolating" languages such as Chinese or Vietnamese. More exactly, there is no language in which all meanings are grammatical /i.e. expressed by grammatical morphs/ or lexical /i.e. expressed by invariant lexical forms/. Hence it makes sense to introduce a new primitive symbol lx denoting the set of all <u>lexical dimensions</u>. Of course, lx should be characterized by some postulates. For instance, it should be assumed that $lx \subset dim$. Further, one should characterize connections between lx and $dim - lx$ /the latter corresponds to grammatical dimensions, as we will see in the moment/.

With the concept of the lexical dimension at hand, one can introduce several further linguistically relevant notions :

7.1. <u>Definition</u>. $grm = dim - lx$. Elements of the set grm are called <u>grammatical dimensions</u>.

7.2. <u>Definition</u>. $lmph = \bigcup lx$, $gmph = \bigcup grm$. Elements of these sets are called <u>proper lexical</u> and <u>proper grammatical morphemes</u>, respectively.

7.3. <u>Definition</u>. For any $x, y \in Lex$ let us define :

$$x \text{ hlx } y \leftrightarrow \mathfrak{A} \cap mp(x) = \mathfrak{A} \cap mp(y) \quad \text{for all } \mathfrak{A} \in lx$$

$$x \text{ hgm } y \leftrightarrow \mathfrak{A} \cap mp(x) = \mathfrak{A} \cap mp(y) \quad \text{for all } \mathfrak{A} \in grm.$$

If x hlx y, then x and y are <u>homolexical</u> and if x hgm y, then x and y are <u>homogrammatical</u>.

Of course, hlx and hgm are both equivalences on Lex.

7.4. <u>Proposition</u>. For any actual lexes x and y, x eq y if and only if x hlx y and x hgm y.

7.5. <u>Proposition</u>. $Lex = \bigcup lmph \cup \bigcup gmph$.

One can define the analogon of the relation of homolexy for lexes :

7.6. <u>Definition</u>. For any $X, Y \in LEX$, let $X \text{ Hlx } Y$ if and only if $MB(X) \cap lmph = MB(Y) \cap lmph$. Let $LXM = LEX/Hlx$. Elements of the set LXM are called <u>lexemes</u>. Let us define the function $LB : LXM \longrightarrow \wp(lmph)$ by $LB(Hlx(X)) = MB(X) \cap lmph$ for any $X \in LEX$. The set $LB(\mathfrak{X})$ is called the <u>lexical bundle</u> of $\mathfrak{X}$.

The concept of lexical bundle can be used for the description of lexical fields, compounds, lexical synonymy, etc.

Let us discuss two more problems connected with the above defined concepts, viz. those of grammatical paradigms and hyponymy extended on the set of lexemes.

7.7. <u>Definition</u>. The function $DM : LEX \longrightarrow \wp(dim)$ is defined for any $x \in Lex$ by $DM(eq(x)) = dm(x)$. The correctness of this definition follows from axiom 6 and proposition 2.20. For any lex $X \in LEX$, let $prd(X)$, the <u>grammatical paradigm</u> of X, be defined as follows :

$$prd(X) = \{Y \in LEX : X \text{ Hlx } Y \wedge DM(X) \cap grm = DM(Y) \cap grm\}.$$

Of course, $prd(X) \subset Hlx(X)$ for any lex $X \in LEX$. Consider an example of the grammatical paradigm of the Finnish lex 'talo' /'house'/ :

prd('talo') = {'talo','talon'/Genetivus/,'talon'/Accusativus/, 'talossa', 'talosta','taloon','talolla','talolta','talolle','talona', 'taloa','taloksi','talotta','taloin'/Instructivus, Singularis/,'taloineen'/Comitativus, Singularis/,'talot'/Nominativus, Pluralis/,'talojen','talot'/Accusativus, Pluralis/,'taloissa','taloista','taloihin','taloilla','taloilta','taloille','taloina','taloja','taloiksi', 'taloitta', 'taloin'/Instructivus, Pluralis/,'taloineen'/Comitativus, Pluralis/}.

7.8. <u>Definition</u>. The relation $HPN \subset LXM^2$ is defined by $\mathfrak{X} \text{ HPN } \mathfrak{Y}$ if and only if $LB(\mathfrak{Y}) \subset LB(\mathfrak{X})$.

7.9. <u>Proposition</u>. HPN is a partial ordering of the set of all lexemes.

The relation HPN may be considered as hyponymy in an abstract sense - if $\mathfrak{X} \text{ HPN } \mathfrak{Y}$, then we ignore everything but lexical information.

The sets lmph and LB[LXM] play a special role with respect to the semantical aspects of our system. It is usually stressed that lexical information points at phenomena /objects, features, processes, qualities, etc./ external with respect to language expressions. In other words, lexical information indicates the denotation of language expressions. In this respect, elements of the set lmph may be considered as denotation primitives and LB[LXM] corresponds to the family of denotations of lexemes. Hence each denotation /of a given lexeme/ is a combination of denotation primitives. One may develop something like a Henkin-style semantics for the lexical universe with the help of the above concepts.

Namely, it is not necessary to introduce any "external" /with respect to language expressions/ objects as denotations of lexemes - the set LB[LXM] forms the required space of denotations. We see here an analogy to the Henkin construction of a model built up from constant symbols /cf. for instance CHANG-KEISLER 1973, ch. 2/.

The investigations concerning the concept of denotation are closely connected with the problem of a definition of <u>signification comprehension</u> of actual meaningful segments. The concept of signification comprehension can be treated as primitive and characterized by suitable postulates. More exactly, let us introduce a new symbol sgc and assume that sgc is a binary relation between eln and the class V of all sets /i.e. we put $V = \{x : x = x\}$/. Further, assume that dom(sgc) = Sgm and denote rng(sgc) = SGC. Finally, let us assume that sgc is a function from Sgm onto SGC and that :

a/ if x P y, then sgc(x) is not contained in sgc(y)

b/ $\{(x,y) : sgc(x) = sgc(y)\} \in FI(Sgm, P\langle Lex\rangle)$

c/ inclusion is finitary upwards in the set SGC.

Then one can <u>define</u> hyponymy hpn by : $x \text{ hpn } y \longleftrightarrow sgc(x) \subset sgc(y)$. Notice that hyponymy defined in this way has all the properties expressed in our axiom system AXIOM 1 - AXIOM 7. Further, it is easy to formulate all the above assumptions concerning sgc in one sentence which will replace axiom 2. In this way we obtain a new axiom system with three primitive terms dim, sgc, hph, equivalent to that presented in sections 2 - 5. Let us call sgc the <u>signification function</u>, sgc(x) the <u>signification comprehension</u> of actual meaningful segment x and SGC the <u>space of all signification comprehensions</u>. Further investigations of these concepts may develop in several directions. For instance, one can look for the connections between sgc and the sets lmph and LB[LXM]. Another problem is that of the internal structure of the set SGC. Using set-theoretical tools one can prove that SGC can be represented as a general system /cf. part III of POGONOWSKI 1979 for the definition of a general system/. Finally, one may replace inclusion in the above definition of hyponymy by some relation between signification comprehensions satisfying suitable formal conditions. The conditions in question should be chosen in such a way that the Frege Principle will be satisfied : signification comprehension of a complex expression should be functionally dependent on signification comprehensions of its parts. The idea of introducing such a relation is similar to investigations taking place in Montague-style semantics /cf. THOMASON 1974/, where λ-calculus is used for the description of the mutual dependencies between extensions and intensions of expressions.

The last extension of the initial axiom system we are going to discuss here concerns moods of expression and the problem of marking. That approach is intended to contain investigations of much more linguistic phenomena than hypotaxis alone. For this reason we have to change our methods - the number of primitive concepts increases, the same concerns

the number of postulates accepted without proof /i.e. it is convenient to give up the method of informal axiomatics in favor of mathematical modelling/. Here we limit ourselves to a few remarks explaining the idea of the extension in question - for more details concerning moods of expression and the problem of marking cf. MAJEWICZ-POGONOWSKI 1983.

In section 2 we developed abstract morphology. Morphemes were treated in an extensional way, as certain classes of actual lexes. Now, the practical question arises whether there are any explicit /material, substantial/ exponents which allow us to put two actual lexes into the same morpheme or into different morphemes. The same question concerns several other units of the plane of expression when those units are grouped together into suitable categories. Briefly, this problem is formulated as the problem of marking : what meanings are expressed in a given language expression and in which a way. The following example from Finnish shows that this problem concerns not only actual lexes but also larger units :

'Hän luki kirjan.'

'Hän luki kirjaa.'

Here ASPECTUS is expressed not by a verb but by the CASUS information concerning the noun playing the role of an object /in the first actual syntagma Perfectivus is expressed by Accusativus ; in the second, Imperfectivus is expressed by Partitivus/. Hence, ASPECTUS should be considered here as a dimension whose morphemes consists not of actual lexes, but of whole actual syntagmas.

There are three sorts of basic objects in our approach : <u>segments</u>, <u>dimensions</u> and <u>moods of expression</u>. Segments are considered as types, not tokens, i.e. as abstract, not concrete /actual/ entities. Segments are categorized into sorts such as texts, sentences, phrases /syntagmas/, morphs, syllables, phones, etc. Dimensions are treated as given a priori: each language has a specific set of lexical and grammatical dimensions. Besides lexical and grammatical dimensions, one can consider some further kinds of dimensions : syntactic, emotive, etc. As before, any dimension is a finite non-empty set of morphemes /meanings/. Finally, moods of expression form a finite non-empty set consisting, among others, of the following elements :

ME1 ORDERING
ME2 AUXILIARIES
ME3 INTONATION
ME4 AFFIXATION
ME5 REDUPLICATION
ME6 COMPOUNDS
ME7 QUANTITY OF A VOWEL
ME8 SUPPLETION
etc.

Elements of this set are treated as abstract entities. The following examples show the use of moods of expression in several languages :

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ME1 : Agens and Patiens in English are determined by the ordering of words - cf. 'John killed Bill' vs. 'Bill killed John'.

ME2 : Auxiliary verbs are used for the purpose of expressing information of TEMPUS /cf. English auxiliary verbs 'to be' and 'to have'/.

ME3 : Meaning can be changed by the different use of a pause /cf. Polish sentences 'Człowiek - z listem przyszedł' and 'Człowiek z listem - przyszedł'/.

ME4 : Affixation is used in almost all /if not all/ languages. Particular languages use different sorts of affixes /for instance, prefixes, postfixes, infixes, circumfixes, etc./.

ME5 : In Malayan, the information of NUMERUS is expressed by reduplication /cf. 'orang' - 'man', 'orang-orang' - 'men'/.

ME6 : In some cases, lexes can be glued together ; then lexical information of one lex may become grammatical information of the compound. Cf. the following Chinese examples :

'kàn' - 'look'	'kànjiàn' - 'have seen'
'jiàn' - 'see'	/act + resultability/
'xué' - 'to learn'	'xuéhǎo' - 'to know'
'hǎo' - 'good'	/act + resultability/.

ME7 : The change of a short vowel for a long one can change the grammatical information /cf. Latin 'venit' - 'he is coming', 'vēnit' - 'he came'/.

ME8 : Suppletion may be used for the expression of GRADATIO QUALITATIS information /cf. English 'good' - 'better'/.

Some information about moods of expression can be found for instance in REFORMATSKIJ 1967 /ch. IV/ and MIEŠČANINOV 1978 /ch. I/.

Several constructions introduced above /such as homolexy, grammatical paradigms, structures in the lexical universe, etc./ can also be made in this approach.

The fundamental problem here is that of mutual connections between segments, dimensions, morphemes, moods of expression and constructs defined by them. Given any language, one can distinguish a set of all <u>admissible</u> <u>triplets</u> as a certain subset of the Cartesian product of the sets of all morphemes, all segments and all moods of expression. Thus, any admissible triplet consists of a meaning /morpheme/ expressed in a segment with the help of a given mood of expression. Several further linguistically relevant concepts can be defined with the help of the concept of an admissible triplet. Moreover, the approach in question can be fully formalized. In this way, we obtain a generalization of our initial system having a much bigger scope of application. Last but not least, this generalization seems to be a convenient starting point for the development of formal typology of natural languages.

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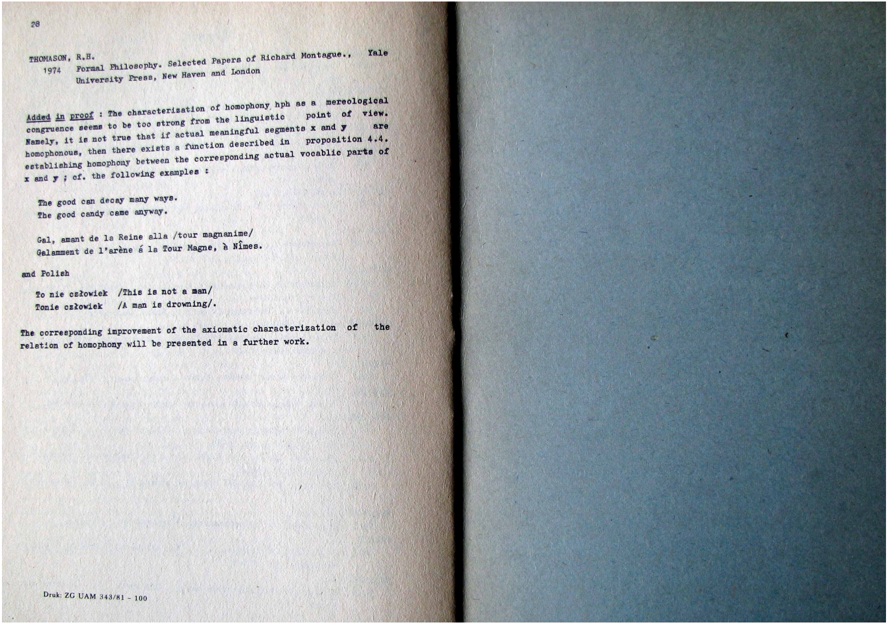

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Added in proof : The characterization of homophony hph as a mereological congruence seems to be too strong from the linguistic point of view. Namely, it is not true that if actual meaningful segments x and y are homophonous, then there exists a function described in proposition 4.4. establishing homophony between the corresponding actual vocablic parts of x and y ; cf. the following examples :

The good can decay many ways.
The good candy came anyway.

Gal, amant de la Reine alla /tour magnanime/
Galamment de l'arène á la Tour Magne, à Nîmes.

and Polish

To nie człowiek /This is not a man/
Tonie człowiek /A man is drowning/.

The corresponding improvement of the axiomatic characterization of the relation of homophony will be presented in a further work.

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