Incompleteness of Effective Double Frames

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By a *double frame* we understand any ordered triple (Φ, Ψ, R) such that Φ and Ψ are distinct non-empty sets and $R \subseteq \Psi \times \Phi$ is a relation with the domain Ψ. For any $x \in \Psi$ let: $R^{\rightarrow}x = \{z \in \Phi : xRz\}$. If Φ and Ψ are subsets of ω (= the set of all natural numbers), then (Φ, Ψ, R) is called a *numerical* double frame.

We say that a double frame (Φ, Ψ, R) is:

- *globally infinite*, if Φ and Ψ are countably infinite sets;
- *locally infinite*, if each set $R^{\rightarrow}x$ is infinite, for all $x \in \Psi$;
- *deeply infinite*, it it is both globally and locally infinite.

We say that a numerical double frame is *effective*, if Φ is recursive, Ψ is r.e. (= recursively enumerable) and R is an r.e. relation. In effective double frames each set $R^{\rightarrow}x$ is r.e., for all $x \in \Psi$. A numerical double frame (Φ, Ψ, R) is *perfect*, if Φ, Ψ, and R are recursive.

Let Δ_Φ be the family of all infinite recursive subsets of a countably infinite recursive set Φ. We say that a globally infinite numerical frame (Φ, Ψ, R) is *recursively complete*, if:

$$\Delta_\Phi \subseteq \{X : X = R^{\rightarrow}x \text{ for some } x \in \Psi\}.$$

The Recursive Jump Theorem. *For any deeply infinite effective double frame (Ψ, Φ, R) there exists an infinite family $\{B_j : j \in \omega\}$ of infinite recursive subsets of Φ such that each B_j is different from any $R^{\rightarrow}x$, for all $x \in \Psi$.*

The Incompleteness Theorem For Effective Double Frames. *No deeply infinite effective double frame is recursively complete.*

Corollary. *No perfect deeply infinite double frame is recursively complete.*

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